Math N55– Practice Midterm 1
Discrete Mathematics

Instructor: Max Hlavacek

July 9, 2019

Name: ____
Student Number: ____________________________________

This exam contains 7 pages (including this cover page) and 5 questions. Total of points is 50.
Good luck !

Distribution of Marks

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

1. Mark each of the following True or False. No explanation required.

   (a) (2 points) The compound proposition $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology.

   (b) (2 points) If x and y are irrational numbers, then $x + y$ is irrational.

   (c) (2 points) For every integer n there is a unique integer m such that $0 \leq m \leq 5$ and $m \equiv n (\bmod 5)$

   (d) (2 points) The set of prime numbers is countably infinite.

   (e) (2 points) If A and B are sets such that $A \subset \mathbb{Z}$ and $B \subset \mathbb{Z}$, then $A \times B = B \times A$.

2. Prove that the following statements are false, i.e. prove their negations.

   (a) (5 points)
   $$\forall a, b \in \mathbb{Z}^+,\ \exists k \in \mathbb{Z}^+(a + bk \text{ is prime}).$$

   (b) (5 points)
   $$\exists a, b \in \mathbb{Z}^+,\ \forall k \in \mathbb{Z}^+(a + bk \text{ is prime}).$$

3. (10 points) Prive that if an integer n is the sum of two squares, then $n \not\equiv 3 \pmod 4$. Here, square means square of an integer.

4. (10 points) Prove that if a and m are positive integers such that $\gcd(a, m) \neq 1$ then a does not have an inverse modulo m.

5. (a) (5 points) Find the remainder when 2^{55} is divided by the prime number 53.

   (b) (5 points) Use the Euclidean Algorithm to find the greatest common divisor of 270 and 63.

This page is intentionally left blank to accommodate work that wouldn't fit elsewhere and/or scratch work.