Math N55– Practice Midterm 2
Discrete Mathematics

Instructor: Max Hlavacek

July 30, 2019

Name: ____

Student ID number: ________________________________

In this exam, you are allowed writing utensils and one 8.5x11 notesheet. No calculators are allowed. Please write your name and id number at the top of each page in the space provided.

This exam contains 7 pages (including this cover page) and 5 questions. The total number of points is 50.
Good luck!

Distribution of Marks

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

1. Mark each of the following True or False. No explanation required. To ease the grading process, please clearly write T for true and F for false on the provided line.

   (a) (2 points) ___ If X and Y are independent random variables, then $E(XY) = E(X)+E(Y)$

   (b) (2 points) ___ If X is a random variable on the sample space S, then $X(s) \geq 0$ for all $s \in S$.

   (c) (2 points) ___ If E and F are independent events in a probability space, then E and $\bar{F}$ are also independent.

   (d) (2 points) ___ Suppose we have holes labeled $0, 1, \ldots n$. If we put $n + 1$ objects in the holes, then at least one hole will contain more than one object.

   (e) (2 points) ___ The recurrence $f_n = f_{n-2}$ where $f(0) = 1$ has exactly one solution.

2. (10 points) A store gives out gift certificates in the amounts of 10 and 25. What amounts of money can you make using gift certificates from the store? Prove your answer using strong induction. Note: You may have to do a finite number of cases separately.

3. Answer the following questions about the set of bit strings of length n.

   (a) (2 points) For $n \geq 0$, determine the number of bit strings of length n that do not contain the string 01.

   (b) (4 points) Let b_n be the number of bit strings of length n that do not contain 000. Show that $b_0 = 1, b_1 = 2$, and $b_2 = 4$ and that

   $$b_{n+3} = b_{n+2} + b_{n+1} + b_n$$

   (c) (4 points) Find the number of strings of length 10 with 4 ones and 6 zeros that do not contain the string 11.

4. (a) (5 points) I have a bag of coins. Nine are fair coins, and the 10th has a heads on both sides. I draw a coin at random and flip it. If the coin comes up heads, what is the probability that the coin was a 2-headed coin?

   (b) (5 points) Let X be a random variable, and $a \in$. Show that $V(aX) = a^2 V(X)$.

5. (10 points) Show using a combinatorial proof that:

$$\binom{2n}{3} = \binom{n}{3} + \binom{n}{3} + \binom{n}{2}n + n\binom{n}{2}$$

This page is intentionally left blank to accommodate work that wouldn't fit elsewhere and/or scratch work.