Representation Theory, Geometry & Combinatorics Seminar

Organizer: M. Haiman and N. Reshetikhin

Wednesday, 4:00–6:00pm, 939 Evans

Jan. 20, 27 **Mike Eastwood**, Australian National Univ., Canberra

Short course: Representation theory and the X-ray transform

The X-ray transform (also called the Funk transform) is obtained by integrating a function or tensor field over the closed geodesics of a Riemannian symmetric space. These lectures will explore the X-ray transform on real and complex projective spaces, employing tools from complex analysis and the representation theory of Lie groups.

Lecture 1: Differential geometry on real and complex projective spaces.

Abstract: As a quotient of the round sphere under the antipodal map, real projective space is a familiar space of constant curvature. With its standard Fubini-Study metric, complex projective space is less familiar but its curvature is also rather simple. Representation theory of the symplectic group can be used to relate these Riemannian manifolds.

Lecture 2: Projective differential geometry

Abstract: Projective differential geometry studies geodesics and, from this point of view, the round sphere is flat. This observation combines with the representation theory of the special linear group (especially the theory of Verma modules) to yield the Bernstein-Gelfand-Gelfand (BGG) complexes of linear differential operators on real projective space.

Lecture 3: The X-ray transform on projective space

Abstract: The BGG complex is just what the doctor ordered to deal with the X-ray transform on real projective space but what about on complex projective space? I shall discuss some substitutes.

Lecture 4: Complex methods

Abstract: Embedding real projective space inside complex projective space allows one to use complex analysis and more tricks from representation theory (especially the Bott-Borel-Weil Theorem) to figure out the range and kernel of the X-ray transform.