

# Representation Theory, Geometry & Combinatorics Seminar

Organizer: M. Haiman and K. Reshetikhin

Wednesday, 4:00–6:00pm, 939 Evans

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May 6      **Philippe Di Francesco**, IPhT Saclay, France and Univ. of Illinois Urbana-Champaign

*Q-systems, paths and cluster positivity*

Q-systems are integrable systems of recursion relations first introduced in the study of quantum spin chains based on Lie groups. These were interpreted recently as mutation relations for certain cluster algebras. The latter were introduced as discrete dynamical systems describing the evolution by mutation of (cluster) variables, with the built-in property that any mutated variable may be expressed as a Laurent polynomial of any other cluster variable. It was conjectured that the corresponding Laurent polynomials have non-negative integer coefficients.

In the present work, we prove this positivity property for the cluster algebras associated to the  $A_r$  Q-systems. This is done with the help of integrability by giving an explicit combinatorial expression for the cluster variables as partition functions for positively weighted paths on finite target graphs. Cluster mutations can then be understood as continued fraction rearrangements for the corresponding generating functions. The weighted graphs are also connected to certain totally positive matrices introduced by Fomin and Zelevinsky in relation to networks. We finally present an alternative formulation in terms of domino tilings of plane domains, including possible defects. This is joint work with R. Kedem.