**Math 55 — Discrete Mathematics — Spring 2003**

Quiz 9 Solutions

Version 1: consider the sequence $g_0, g_1, \ldots$ defined recursively by

$$g_0 = 0$$
$$g_1 = 1$$
$$g_n = g_{n-1} + 2g_{n-2} \quad \text{for } n \geq 2.$$

Use strong induction to prove that $g_n < 2^n$ for all $n \geq 0$.

Version 2: consider the sequence $g_0, g_1, \ldots$ defined recursively by

$$g_0 = 0$$
$$g_1 = 1$$
$$g_n = 2g_{n-1} + 3g_{n-2} \quad \text{for } n \geq 2.$$

Use strong induction to prove that $g_n < 3^n$ for all $n \geq 0$.

*Proof* (version 1): For $n = 0$ and $n = 1$, the desired result holds, since $0 < 1 = 2^0$ and $1 < 2 = 2^1$. For $n \geq 2$, assume by induction that the result holds for all $0 \leq k < n$. Then

$$\begin{aligned} g_n &= g_{n-1} + 2g_{n-2} \quad \text{(by definition)} \\ &< 2^{n-1} + 2 \cdot 2^{n-2} \quad \text{(by induction)} \\ &= 2^{n-1} + 2^{n-1} = 2^n. \end{aligned}$$

*Proof* (version 2): For $n = 0$ and $n = 1$, the desired result holds, since $0 < 1 = 3^0$ and $1 < 3 = 3^1$. For $n \geq 2$, assume by induction that the result holds for all $0 \leq k < n$. Then

$$\begin{aligned} g_n &= 2g_{n-1} + 3g_{n-2} \quad \text{(by definition)} \\ &< 2 \cdot 3^{n-1} + 3 \cdot 3^{n-2} \quad \text{(by induction)} \\ &= 2 \cdot 3^{n-1} + 3^{n-1} = 3^n. \end{aligned}$$