

Math 55 — Discrete Mathematics — Spring 2003

Quiz 6 Solutions

Consider the linear code C over \mathbb{Z}_2 with 3 message bits, 10 code bits, and code matrix

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}.$$

(a) Encode the message vector $[1 \ 0 \ 1]$.

$$[1 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0]$$

(b) Find $w(C)$, the minimum weight of a non-zero code vector.

Each row of the code matrix has weight 5. Any sum of two rows will have four 1's in the first six columns and two 1's in the last four columns, for a weight of 6. The sum of all three rows is $[1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 1]$, which has weight 7. So the minimum weight is 5.

(c) How many errors can the code C correct?

$w(C) = 5 = 2e + 1$ with $e = 2$. So C can correct up to two errors.

(d) How many message bits can be encoded by a simple redundancy code with ten or fewer code bits and the ability to correct as many errors as the code C ?

To correct two errors, you need five-fold redundancy, so you could only encode two message bits in 10 code bits. Since the code C encodes three message bits with the same error-correction capability, it is superior to a simple redundancy code.