

Math 55 — Discrete Mathematics — Spring 2003

Quiz 5 Solutions

(a) Express the system of equations $2x - y + 4z = -2$, $3x - 2y + 2z = -1$ (version 1) or $3x + 4y + 2z = 2$, $2x + 3y - z = -3$ (version 2) as a matrix equation $\mathbf{Ax} = \mathbf{b}$.

Version 1:

$$\begin{bmatrix} 2 & -1 & 4 \\ 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} x & y & z \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

Version 2:

$$\begin{bmatrix} 3 & 4 & 2 \\ 2 & 3 & -1 \end{bmatrix} \begin{bmatrix} x & y & z \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

(b) Use row operations to find an equivalent system $\mathbf{A}'\mathbf{x} = \mathbf{b}'$ in which the first columns of \mathbf{A}' form an identity matrix.

Version 1:

$$\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 8 \end{bmatrix} \begin{bmatrix} x & y & z \end{bmatrix} = \begin{bmatrix} -3 \\ -4 \end{bmatrix}$$

Version 2:

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & -7 \end{bmatrix} \begin{bmatrix} x & y & z \end{bmatrix} = \begin{bmatrix} 18 \\ -13 \end{bmatrix}$$

(c) Find all solutions of the original system of equations, and the solution with $z = -1$ (version 1) or with $z = 2$ (version 2).

Version 1: $x = -6z - 3$, $y = -8z - 4$; with $z = -1$, we get $x = 3$, $y = 4$.

Version 2: $x = -10z + 18$, $y = 7z - 13$; with $z = 2$, we get $x = -2$, $y = 1$.