Math 55 — Discrete Mathematics — Spring 2003

Quiz 12 Solutions

Problem 1. (Version 1) How many ways are there to pay 6 people a total of 100 dollars, if everyone must get at least 10 dollars (and everyone gets an integer number of dollars)?

The problem is equivalent to paying 40 dollars to 6 people without the 10 dollar minimum, or counting non-negative integer compositions

$$x_1 + x_2 + \cdots + x_6 = 40.$$

The number of these is

$$\binom{6+40-1}{40}.$$

Problem 1. (Version 2) How many ways are there to pay 5 people a total of 100 dollars, if everyone must get at least 15 dollars (and everyone gets an integer number of dollars)?

$$\binom{5+25-1}{25}$$

Problem 2. (Version 1) Prove that if X is any 8-element subset of $\{1, 2, \ldots, 15\}$, then there exist elements $w, x, y, z \in X$ such that $x + y = w + z$ and $\{x, y\} \neq \{w, z\}$.

The smallest possible sum is $1+2 = 3$; the largest is $14+15 = 29$. We have $\binom{8}{2} = 28$ pairs $\{x, y\} \subseteq X$, and their sums belong to the set $\{3, 4, \ldots, 29\}$, which has $29-2 = 27$ elements. By the pigeonhole principle, two of the sums must be equal.

Problem 2. (Version 2) Prove that if X is any 7-element subset of $\{1, 2, \ldots, 11\}$, then there exist elements $w, x, y, z \in X$ such that $x + y = w + z$ and $\{x, y\} \neq \{w, z\}$.

Same reasoning as in Version 1, except now we have $\binom{7}{2} = 21$ pairs, with sums in the set $\{3, 4, \ldots, 21\}$, which has 19 elements.