Quiz 1 Solutions

There were two versions of the quiz.

Version **A**:

Determine whether each of the following functions is $O(n \log n)$, whether it is $\Omega(n \log n)$ and whether it is $\Theta(n \log n)$:

(a) $n \log(n^2) + n^2 \log n$

(b) $(n + \log n)(\log n)$

Solution:

(a) Since $(n^2 \log n)/(n \log n) = n$, which goes to infinity, the second term alone is $\Omega(n \log n)$ and not $O(n \log n)$. The function as a whole is bigger than its second term, so it is $\Omega(n \log n)$ but not $O(n \log n)$ and therefore also not $\Theta(n \log n)$.

(b) The first factor is $\Theta(n)$ and the second is $\Theta(\log n)$, so their product is $\Theta(n \log n)$ and therefore also $O(n \log n)$ and $\Omega(n \log n)$.

Version **B**:

Determine whether each of the following functions is $O((\log x)^2)$, whether it is $\Omega((\log x)^2)$ and whether it is $\Theta((\log x)^2)$:

(a) $\log(x^2) \log(x^3)$

(b) $(x + \log x)(\log x)$

Solution:

(a) Both factors are $\Theta(\log x)$, since the first is equal to $2 \log x$ and the second to $3 \log x$. Hence their product is $\Theta((\log x)^2)$ and therefore also $O((\log x)^2)$ and $\Omega((\log x)^2)$.

(b) The first factor is $\Theta(x)$ and the second is $\Theta(\log x)$, so the product is $\Theta(x \log x)$. Since $(x \log x)/((\log x)^2) = x/(\log x)$ goes to infinity as $x \to \infty$, we see that $x \log x$ is $\Omega((\log x)^2)$, not $O((\log x)^2)$ and not $\Theta((\log x)^2$. The same holds true for the given function $(x + \log x)(\log x)$, since it is $\Theta(x \log x)$.