Homework 14 Solutions

4.4 #22: (a) Both sides count pairs of subsets $A \subseteq B$ of an n-element set X, where $|B| = r$ and $|A| = k$. For the left side, choose B first, then A. For the right side, choose A first, then choose $B \setminus A$ from $X \setminus A$. The last choice is of $r - k$ elements from a set of $n - k$.

(b) The formula for each side simplifies to $\frac{n!}{(n-r)!(r-k)!k!}$. Note that this is also equal to the multinomial coefficient $\binom{n}{n-r, r-k, k}$.

4.5 #42-43: There are $\binom{52}{13,13,13,13}$ deals. To count the deals with an ace in each hand, distribute the aces 4! ways, then deal the rest as four hands of 12, for a total of $4!\binom{48}{12,12,12,12}$ ways. The probability of dealing one ace in each hand is

$$4!\binom{48}{12,12,12,12} \Big/ \binom{52}{13,13,13,13} \sim 0.105$$

Note that the answer to #43 in the book is the probability, not the number of possible deals.

4.2 #10: The midpoint of two integer points (x_1, y_1) and (x_2, y_2) is $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$. This is an integer point if and only x_1 and x_2 are both odd or both even, and also y_1 and y_2 are both odd or both even.

Given 5 points, the pigeonhole principle implies that there are 3 with x-coordinates all odd or all even. Applying the pigeonhole principle to these 3, among them must be two with y-coordinates both odd or both even. These two are our desired points.