Homework 10 Solutions

#6: The desired identity is

$$\frac{1}{1\cdot 2} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}.$$

Proof by induction on n. Basis step is $n = 1$, where both sides reduce to $1/2$. For the induction step, assume

$$\frac{1}{1\cdot 2} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}.$$

Add $1/(n+1)(n+2)$ to both sides to get

$$\begin{aligned}\frac{1}{1\cdot 2} + \cdots + \frac{1}{n(n+1)} + \frac{1}{(n+1)(n+2)} &= \frac{n}{n+1} + \frac{1}{(n+1)(n+2)} \\ &= \frac{n(n+2)+1}{(n+1)(n+2)} \\ &= \frac{(n+1)^2}{(n+1)(n+2)} \\ &= \frac{n+1}{n+2}.\end{aligned}$$

#34 uses strong induction and is one of the problems deferred to next week, while #43 was supposed to be for this week. My apologies for not correcting the web page earlier.

#52: The inductive step doesn't follow because $x-1$ and $y-1$ are not necessarily positive integers.