Homework problems for Lecture 20

1. Call a skew shape *anti-normal* if it is a 180° rotation of a normal (non-skew) shape. Equivalently, a skew shape is anti-normal if it can be written as λ/μ, where λ is a rectangle.

(a) Show that if X has antinormal shape and $T \sqcup X$ makes sense, then $J^X(T)$ has antinormal shape and depends only on T, not on X. Denote this tableau by $J^\square(T)$.

(b) Show that if T has normal shape λ, then the shape of $J^\square(T)$ is the 180° rotation of λ. [Hint: using dual equivalence, it suffices to do this for one specially chosen T of the given shape.]

(c) Schützenzerger's *evacuation* operator $T \to \mathrm{ev}(T)$ is defined as follows. Given T of normal shape, $\mathrm{ev}(T)$ is the tableau obtained from $J^\square(T)$ by rotating it 180° and renumbering the entries in reverse order, $1, 2, \ldots, n \mapsto n, \ldots, 2, 1$. Show that evacuation is an involution.

2. Consider the following sequence of operations on a tableau T of normal shape λ:

(1) delete the smallest entry;

(2) perform a jeu-de-taquin slide into the now empty cell $(0, 0)$;

(3) repeat steps (1) and (2) until all entries have been removed;

(4) form the tableau S whose entries $n, \ldots, 2, 1$ occupy the cells of λ in the order they are vacated by step (2).

Show that the resulting tableau S is equal to the evacuation $\mathrm{ev}(T)$, as defined in Problem 1.