Math 1A—Calculus, Fall 2010—Haiman
Final Exam

1. [4] Simplify $\cos(\sin^{-1}(2x))$ and state its domain.

2. [4] At which points is the function $f(x) = \begin{cases} 1/(1-e^x) & x < 0 \\ -x & 0 \le x < 1 \\ \cos \pi x & x \ge 1 \end{cases}$ (a) continuous, (b) continuous from the right, (c) continuous from the left, (d) neither?

3. [4] Differentiate $\ln(e^{\sqrt{2}\,x} + e^{-\sqrt{2}\,x})$.

4. [5] Find the point (a, b) on the graph $y = e^x$ where its tangent line passes through $(0, 0)$.

5. [4] Use a linear approximation or differentials to estimate $(8.15)^{2/3}$.

6. [4] Find the limit, either finite or infinite, or explain why it does not exist.

$$\lim_{x \to \pi/2} \frac{e^x - 1}{\cos x}$$

7. [4] Find the limit, either finite or infinite, or explain why it does not exist.

$$\lim_{x \to \infty} \frac{\ln(x+2)}{\ln(x+1)}$$

8. [4] Find the limit, either finite or infinite, or explain why it does not exist.

$$\lim_{x \to 1} (x+1)^{x-1}$$

9. [5] If $x^3 + y^3 = xy + 2$, find dy/dx in terms of x and y.

10. [5] If $-1 \le f'(x) \le 1$ for all x, and $f(1) = 5$, what can you conclude about the value of $f(4)$?

11. [5] Find all local maxima and minima of the function $f(x) = \dfrac{x}{x^2+9}$.

12. [5] Find the largest possible perimeter of a rectangle with lower-left corner at $(0, 0)$ and upper-right corner on the arc of the curve $xy = 4$ between $(1, 4)$ and $(4, 1)$.

13. [5] Alice is walking east and Bob is walking west along opposite sides of a street 10 m wide. If each walks at a speed of 2 m/s, how fast is the distance between them decreasing when Alice is 30 m west of Bob?

14. [4] For what values of A is the graph of $\cos x + Ax^2$ concave upward at every point?

15. [4] Show that $\displaystyle\int (\ln x)^2\,dx = x(\ln x)^2 - 2\,x \ln x + 2\,x + C$.

16. [5] Evaluate the integral $\int_0^2 |x(x-1)|\,dx$.

17. [5] Evaluate the indefinite integral $\int \frac{x^3}{x^2+1}\,dx$.

18. [5] Evaluate the integral $\int_0^{\pi/4} \tan x\,dx$.

19. [5] Find the area of the region enclosed by the curve $xy = 3$ and the line $x + y = 4$.

20. [5] Let R be the region bounded by the x-axis, the line $x = e$, and the graph of $y = \ln x$. Set S be the solid obtained by rotating R about the y-axis.

Set up, but do not evaluate, an integral which gives the volume of S using the method of slices.

21. [5] Set up, but do not evaluate, an integral which gives the volume of the solid S in the previous problem using the method of cylindrical shells.

22. [4] Find the average value of $\sqrt{1-x^2}$ on the interval $[-1, 1]$.