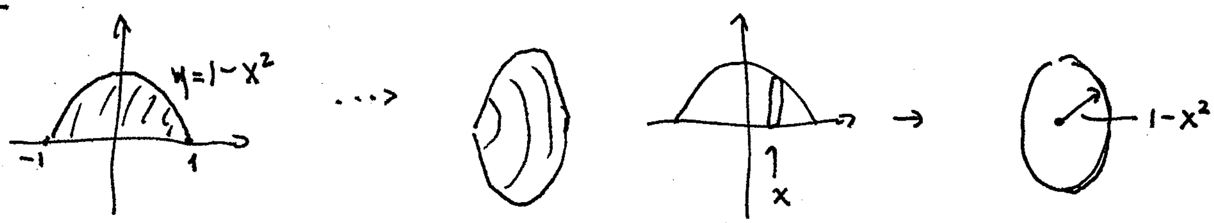
Math 1A – Fall 2010 – Haiman

HW 14 Solutions

6.2 2.

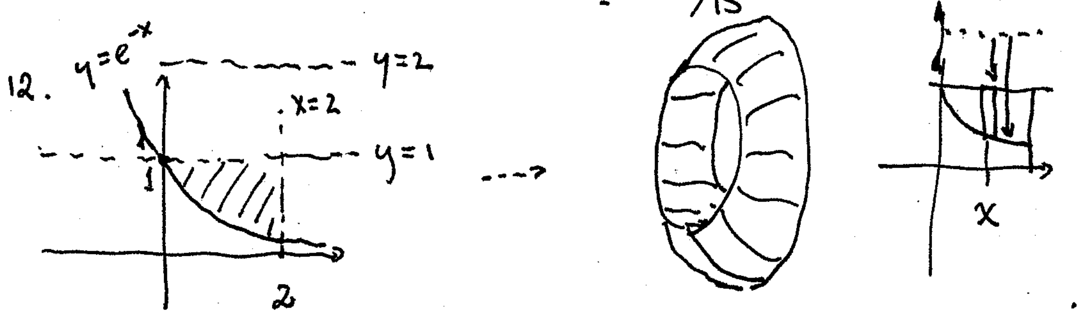


$$V=\int_{-1}^{1}\pi(1-x^2)^2\,dx = \pi\int_{-1}^{1}x^4-2x^2+1\,dx$$

$$=2\pi\int_0^1 x^4-2x^2+1\,dx = 2\pi\left(\frac{x^5}{5}-\frac{2x^3}{3}+x\right)\Big]_0^1$$

$$=16\pi/15$$

12.



· inner radius = 1

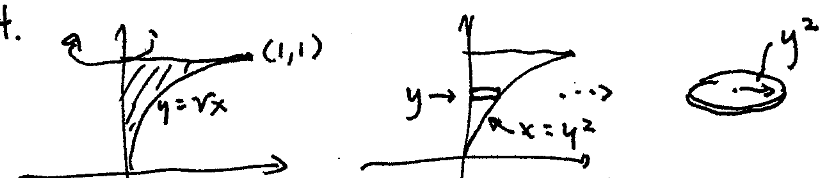
· outer radius $= 2-e^{-x}$

$$V=\int_0^2 \pi(2-e^{-x})^2-\pi\cdot 1^2\,dx$$

$$=\int_0^2 \pi e^{-2x}-4\pi e^{-x}+3\pi\,dx$$

$$=-\frac{\pi e^{-2x}}{2}+4\pi e^{-x}+3\pi x\Big]_0^2 = 4\pi e^{-2}-\frac{\pi}{2}e^{-4}+6\pi$$

24.

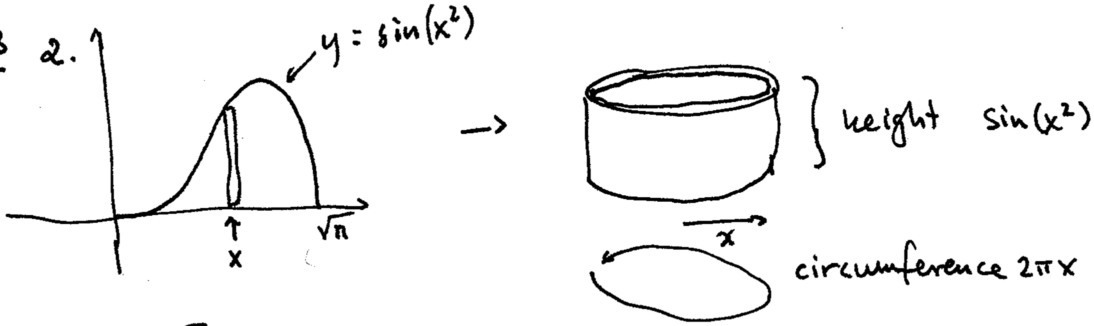


$$V=\int_0^1\pi(y^2)^2\,dy=\int_0^1\pi y^4=\pi y^5/5\Big]_0^1=\frac{\pi}{5}$$

This gives $V = \int_{-r}^{r} 4(r^2 - z^2)\,dz = 4r^2 z - \frac{4}{3}z^3\Big]_{-r}^{r}$

$= \frac{8}{3}r^3 - \left(-\frac{8}{3}r^3\right) = \frac{16r^3}{3}$.

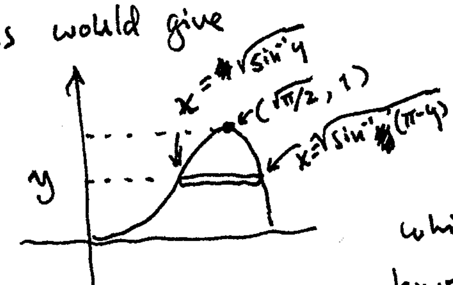
6.3 2.



$V = \int_0^{\sqrt{\pi}} 2\pi x \sin(x^2)\,dx$ $\qquad u = x^2 \qquad du = 2x\,dx$

$\int_0^{\pi} \pi \sin u\,du = -\pi\cos u\Big]_0^{\pi} = 2\pi$

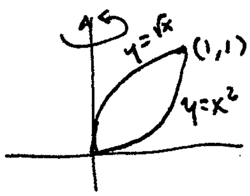
Slices would give



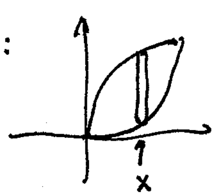
$\int_0^1 \pi \sin^{-1}(\pi - y) - \pi \sin^{-1} y\,dy$,

which is tricky to set up, and we don't know a good techique for evaluating the integral.

8.



Shells:

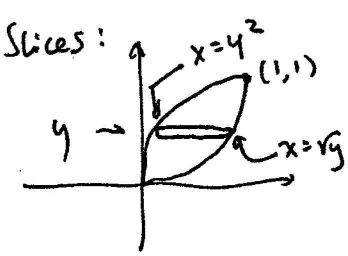


$\int_0^1 2\pi x\,(\sqrt{x} - x^2)\,dx$

$= \int_0^1 2\pi x^{3/2} - 2\pi x^3\,dx$

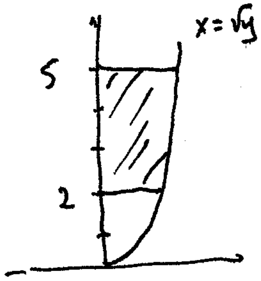
$= 2\pi\left[\frac{2}{5}x^{5/2} - \frac{1}{4}x^4\right]_0^1 = 3\pi/10$

Slices:



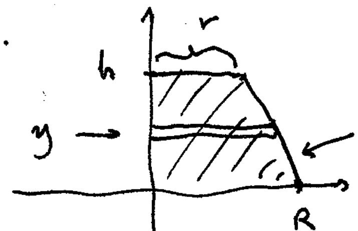
$\int_0^1 \pi y - \pi y^4\,dy = \pi\left(\frac{y^2}{2} - \frac{y^5}{5}\right)\Big]_0^1 = 3\pi/10$

42. $x = \sqrt{y}$, $y = x^2$



Solid of rotation of the region between the parabola $y = x^2$, the y-axis, and the lines $x = 2$, $x = 5$, about the y-axis.

50.



$x = r + \frac{R-r}{h}(h-y) = R - \frac{R-r}{h} y$

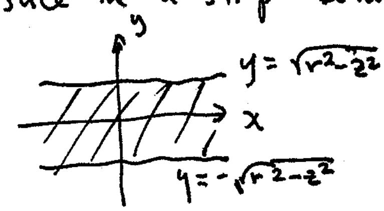
$$V = \int_0^h \pi \left(R - \frac{R-r}{h} y\right)^2 dy \qquad u = R - \frac{R-r}{h} y \qquad du = -\frac{R-r}{h} dy$$

$$= -\frac{h}{R-r}\int_R^r \pi u^2 \, du = \frac{h}{R-r}\int_r^R \pi u^2 \, du = \frac{\pi h}{R-r} u^3/3 \Big]_r^R =$$

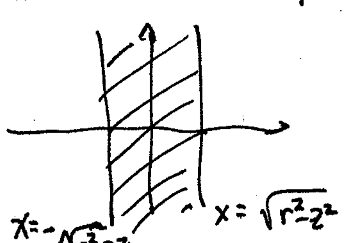
$$\frac{\pi h}{3} \, \frac{R^3 - r^3}{R-r} = \frac{\pi h (R^2 + rR + r^2)}{3}$$

66. Use slices $\perp$ to the z-axis.

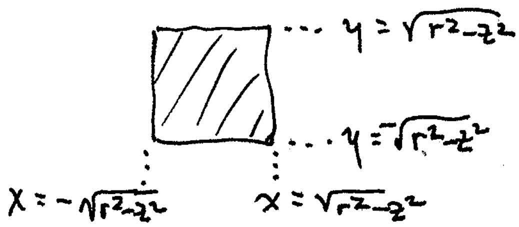
The cylinder about the x-axis is $y^2 + z^2 = r^2$, so it meets the z-slice in a strip between $y = \pm\sqrt{r^2 - z^2}$:



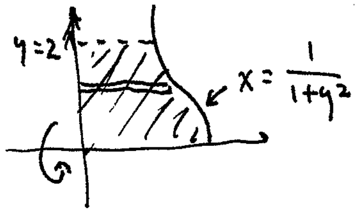
Similarly, the cylinder about the y-axis, $x^2 + z^2 = r^2$ meets the z-slice in a strip:

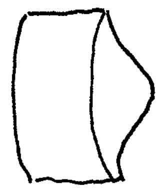


So the intersection of the two cylinders leaves z-slice a square of side length $2\sqrt{r^2 - z^2}$.



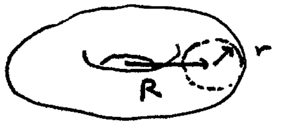
30.

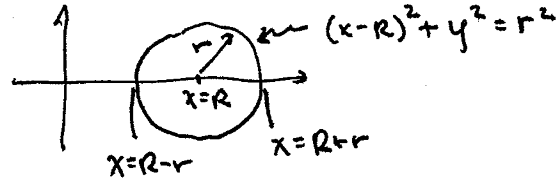


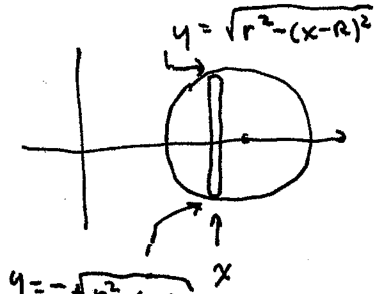


Solid of rotation of the region bounded by the x- and y- axes, the line $y=2$, and the curve $x=\frac{1}{1+y^2}$, about the x-axis.

44. (Also done in lecture)







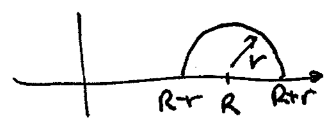
$$V=\int_{R-r}^{R+r} 2\pi x \cdot 2\sqrt{r^2-(x-R)^2}\,dx$$

$u=r^2-(x-R)^2 \qquad du = 2(x-R)\,dx$

$$V=\int_{R-r}^{R+r} 4\pi(x-R)\sqrt{r^2-(x-R)^2}\,dx+\int_{R-r}^{R+r}4\pi R\sqrt{r^2-(x-R)^2}\,dx$$

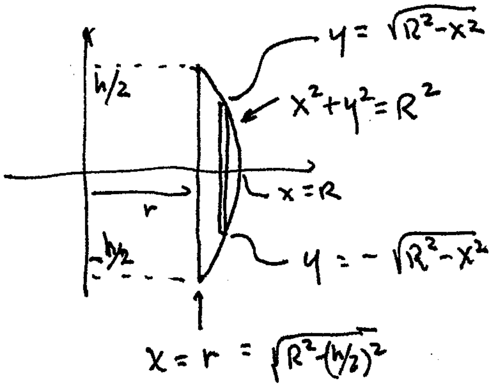
$$=\int_0^0 2\pi\sqrt{u}\,du+\int_{R-r}^{R+r}4\pi R\sqrt{r^2-(x-R)^2}\,dx.$$

The first $\int$ is 0. The second is $4\pi R\underbrace{\int_{R-r}^{R+r}\sqrt{r^2-(x-R)^2}\,dx}_{=\text{ area of semicircle}}$



So we get $V=4\pi R\cdot\frac{\pi r^2}{2}=2\pi^2Rr^2$

46. (Also done in lecture)



$$V = \int_r^R 2\pi x \cdot 2\sqrt{R^2 - x^2}\, dx$$

$u = R^2 - x^2 \qquad du = -2x\, dx$

$$-\int_{R^2-r^2}^{0} 2\pi \sqrt{u}\, du = \int_0^{R^2-r^2} 2\pi \sqrt{u}\, du$$

$$= \frac{4\pi}{3} u^{3/2} \Big]_0^{R^2-r^2} = \frac{4\pi}{3} (R^2 - r^2)^{3/2}.$$

But $R^2 - r^2 = \left(\frac{h}{2}\right)^2$, so this is equal to $\frac{4\pi}{3}\left(\frac{h}{2}\right)^3 = \frac{\pi h^3}{6}$, independent of R.

6.5 2. $\frac{1}{2\pi}\int_{-\pi}^{\pi} \sin 4x\, dx = 0$ (since $\sin 4x$ is an odd function).

14. The average is $\frac{1}{b}\int_0^b 2 + 6x - 3x^2\, dx = \frac{1}{b}\left(2x + 3x^2 - x^3\right)\Big]_0^b$

$$= \frac{1}{b}(2b + 3b^2 - b^3) = 2 + 3b - b^2$$

Solving $2 + 3b - b^2 = 3$: $b^2 - 3b + 1 = 0$;

$$b = \frac{3 \pm \sqrt{5}}{2}.$$

24. $f_{av}[a,b] = \frac{1}{b-a}\int_a^b f(x)\, dx = \frac{1}{b-a}\left(\int_a^c f(x)\, dx + \int_c^b f(x)\, dx\right)$

$$= \frac{1}{b-a}\left((c-a) f_{av}[a,c] + (b-c) f_{av}[b,c]\right)$$

$$= \frac{c-a}{b-a} f_{av}[a,c] + \frac{b-c}{b-a} f_{av}[b,c]$$