Math 1A - Fall 2010 - Haiman

HW 13 Solutions

6.1 2. $\displaystyle\int_0^2 \sqrt{x+2} - \frac{1}{x+1}\,dx = \int_0^2 \sqrt{x+2}\,dx - \int_0^2 \frac{1}{x+1}\,dx$

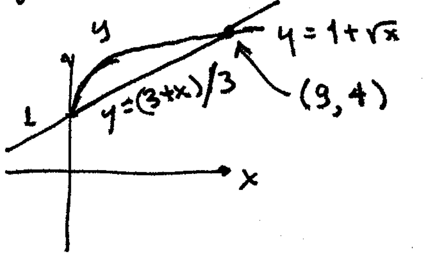
$$\int_0^2 \sqrt{x+2}\,dx = \underset{u=x+2,\ du=dx}{\int_2^4 u^{1/2}\,du} = \frac{2}{3}u^{3/2}\Big]_2^4 = \frac{16}{3} - \frac{2\sqrt{2}}{3}$$

$$\int_0^2 \frac{1}{x+1}\,dx = \underset{u=x+1,\ du=dx}{\int_1^3 \frac{1}{u}\,du} = \ln u\Big]_1^3 = \ln 3.$$

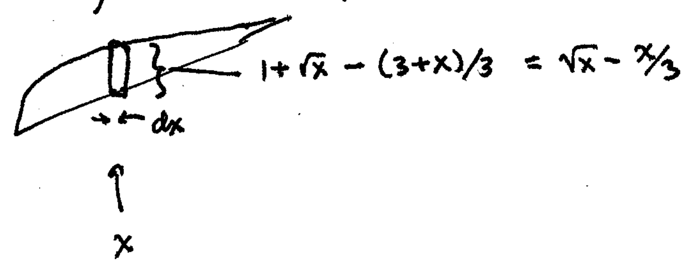
$$A = \frac{16-2\sqrt{2}}{3} - \ln 3.$$

4. $\displaystyle\int_0^3 (2y-y^2)-(y^2-4y)\,dy = \int_0^3 6y-2y^2\,dy = 3y^2 - \frac{2y^3}{3}\Big]_0^3 = 27-18=9$

10.

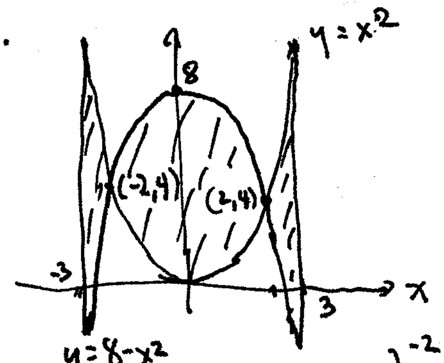


Using vertical strips



$$A = \int_0^9 \sqrt{x} - \frac{x}{3}\,dx = \frac{2}{3}x^{3/2} - \frac{x^2}{6}\Big]_0^9 = \frac{2}{3}\cdot 27 - \frac{81}{6} = 18 - \frac{27}{2} = \frac{9}{2}$$

18.



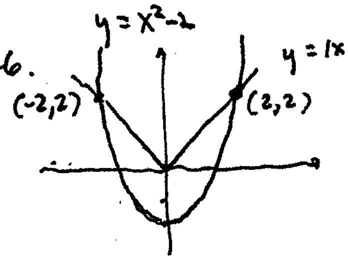
$$\int_{-3}^{-2} x^2-(8-x^2)\,dx + \int_{-2}^{2} (8-x^2)-x^2\,dx + \int_2^3 x^2-(8-x^2)\,dx$$

$$= \int_{-3}^{-2} 2x^2-8\,dx + \int_{-2}^{2} 8-2x^2\,dx + \int_2^3 2x^2-8\,dx$$

$$= \frac{2x^3}{3} - 8x\Big]_{-3}^{-2} + 8x - \frac{2x^3}{3}\Big]_{-2}^{2} + \frac{2x^3}{3} - \frac{8}{x}\Big]_2^3$$

$=$

26.



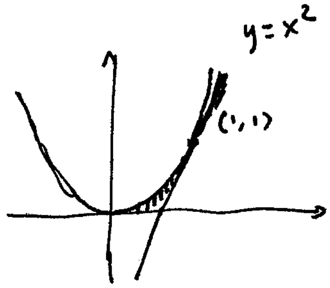
$$\int_{-2}^{2} |x| - (x^2 - 2)\, dx \underset{\uparrow}{=} 2\int_0^2 x - (x^2 - 2)\, dx$$

by symmetry

$$= 2\left(-\frac{x^3}{3} + x^2 + 2x\right]_0^2\bigg) = \frac{32}{3}$$

42. $2 \cdot (3.1 + 6.7 + 7.0 + 6.2 + 5.3 + 4.9 + 4.8 + 2.4)$

48.



$y' = 2x$: at $x = 1$, $y' = 2$, so the tangent line is $y = 2(x-1) + 1 = 2x - 1$.
It meets the x-axis at $x = 1/2$.

It's easiest to use horizontal strips to find the area; otherwise you have to split the integral into two pieces since the lower boundary is different on $[0, \frac{1}{2}]$ and on $[1/2, 1]$. The tangent line is $x = \frac{y+1}{2}$.

So: $A = \displaystyle\int_0^1 \frac{y+1}{2} - \sqrt{y}\, dy$

$$= \frac{1}{2}y + \frac{1}{4}y^2 - \frac{2}{3}y^{3/2}\Big]_0^1 = \frac{1}{12}$$