Math 1A - Fall 2010 - Haiman
HW 12 Solutions
5

5.5 2. $\int x^3 (2+x^4)^5 \, dx$ $\quad u = 2+x^4$, $du = 4x^3 \, dx$

$$= \frac{1}{4}\int u^5 \, du = \frac{u^6}{24} + C = \frac{(2+x^4)^6}{24} + C$$

8. $\int x^2 (x^3+5)^9 \, dx$ $\quad u = x^3+5$ $\quad du = 3x^2 \, dx$

$$= \frac{1}{3}\int u^9 \, du = \frac{u^{10}}{30} + C = \frac{(x^3+5)^{10}}{30} + C$$

16. $\int \frac{x}{x^2+1} \, dx$ $\quad u = x^2+1$ $\quad du = 2x \, dx$

$$= \frac{1}{2}\int \frac{1}{u} \, du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln(x^2+1) + C$$ ← can drop $|\cdot|$ since $x^2+1 > 0$

18. $\int \sec 2\theta \tan 2\theta \, d\theta$ $\quad u = 2\theta$ $\quad du = 2 d\theta$

$$= \frac{1}{2}\int \sec u \tan u \, du = \frac{1}{2}\sec u + C = \frac{1}{2}\sec 2\theta + C$$

30. $\int \frac{\sin(\ln x)}{x} \, dx$ $\quad u = \ln x$ $\quad du = \frac{1}{x} dx$

$$= \int \sin u \, du = -\cos u + C = -\cos \ln x + C$$

46. $\int x^3 \sqrt{x^2+1} \, dx$ $\quad u = x^2+1$ $\quad du = 2x \, dx$

$x^2 = u-1$

$$= \frac{1}{2}\int (u-1)\sqrt{u} \, du = \frac{1}{2}\int u^{3/2} - u^{1/2} \, du = \frac{u^{5/2}}{5} - \frac{u^{3/2}}{3} + C$$

$$= \frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + C$$

52. $\int_0^7 \sqrt{4+3x} \, dx$ $\quad u = 4+3x$ $\quad du = 3dx$

$$= \frac{1}{3}\int_4^{25} \sqrt{u} \, du = \frac{1}{3}\left.\frac{u^{3/2}}{3/2}\right]_4^{25} = \frac{2}{9}\left.u^{3/2}\right]_4^{25} = \frac{2}{9}(125-8) = \frac{2\cdot 117}{9} = 26$$

58. $\int_0^1 x e^{-x^2}\,dx$ $\qquad u = -x^2 \quad du = -2x\,dx$

$= -\frac{1}{2}\int_0^{-1} e^u\,du = -\frac{1}{2}e^u\Big]_0^{-1} = -\frac{1}{2}e^{-1} + \frac{1}{2} = \frac{1}{2}(1-e^{-1})$

68. $\int_0^{1/2} \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$ $\qquad u = \sin^{-1}x \quad du = \frac{1}{\sqrt{1-x^2}}\,dx$

$= \int_0^{\pi/6} u\,du = \frac{u^2}{2}\Big]_0^{\pi/6} = \frac{\pi^2}{72}$

82. $\int_0^3 x\,f(x^2)\,dx$ $\qquad u = x^2 \quad du = 2x\,dx$

$= \frac{1}{2}\int_0^9 f(u)\,du = \frac{1}{2}\cdot 4 = 2.$

(4 given)