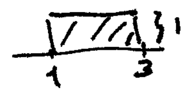
Math 1A – Fall 2010 – Haiman
HW 11 Solutions

5.2 44. $\int_1^3 (2e^x - 1)\,dx = 2\int_1^3 e^x dx - \int_1^3 1\,dx$

$= 2(e^3 - e) - 2$

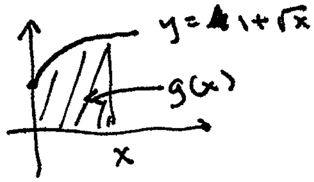
By Ex. 3 ↑   ↑ area of 

54. $\cos x$ is decreasing on $[\pi/6, \pi/4]$, so its minimum on this interval is $\cos \pi/4 = \frac{\sqrt{2}}{2}$ and its maximum is $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$.

Hence $\int_{\pi/6}^{\pi/4} \frac{\sqrt{2}}{2}\,dx \le \int_{\pi/6}^{\pi/4} \cos x\,dx \le \int_{\pi/6}^{\pi/4} \frac{\sqrt{3}}{2}\,dx$

$\int_{\pi/6}^{\pi/4} \frac{\sqrt{2}}{2}\,dx = \frac{\sqrt{2}}{2}(\pi/4 - \pi/6) = \frac{\sqrt{2}\pi}{24}$

$\int_{\pi/6}^{\pi/4} \frac{\sqrt{3}}{2}\,dx = \frac{\sqrt{3}}{2}(\pi/4 - \pi/6) = \frac{\sqrt{3}\pi}{24}$

5.3 6. 

a) By FTC, $g'(x) = 1 + \sqrt{x}$

b) $g(x) = \int_0^x 1 + \sqrt{t}\,dt = t + \frac{t^{3/2}}{3/2}\Big]_0^x$

$= x + \frac{x^{3/2}}{3/2}$,

so $g'(x) = 1 + x^{1/2} = 1 + \sqrt{x}$

12. $G(x) = \int_x^1 \cos\sqrt{t}\,dt = -\int_1^x \cos\sqrt{t}\,dt \Rightarrow G'(x) = -\cos\sqrt{x}$

14. $h(x) = \int_0^{x^2} \sqrt{1+r^3}\,dr = g(x^2)$ where $g(x) = \int_0^x \sqrt{1+r^3}\,dr$

$h'(x) = g'(x^2)\cdot 2x = \sqrt{1+x^3}\cdot 2x$

26. $\int_\pi^{2\pi} \cos\theta\,d\theta = \sin\theta\Big]_\pi^{2\pi} = 0 - 0 = 0$

36. $\int_0^1 10^x\,dx = \frac{10^x}{\ln 10}\Big]_0^1 = \frac{10}{\ln 10} - \frac{1}{\ln 10} = \frac{9}{\ln 10}$

40. $\int_1^2 \frac{4+u^2}{u^3}\,du = \int_1^2 4u^{-3} + u^{-1}\,du = -2u^{-2} + \ln u\Big]_1^2 = -\frac{1}{2} + \ln 2 - (-2 + \overset{0}{\ln 1})$

$= \frac{3}{2} + \ln 2$

58. $y' = \frac{1}{1+x+x^2}$ by FTC;

$y'' = \frac{-1}{(1+x+x^2)^2} \cdot (2x+1) > 0$ for $x < -\frac{1}{2}$, i.e. on $(-\infty, -\frac{1}{2})$

5.4 2. $\frac{d}{dx}(x \sin x + \cos x + C) = \sin x + x \cos x - \sin x = x \cos x$,

so $\int x \cos x \, dx = x \sin x + \cos x + C$

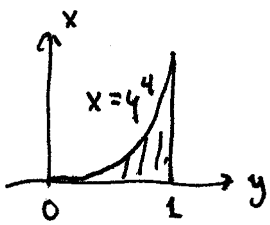
16. $\int \sec t (\sec t + \tan t)\, dt = \int \sec^2 t \, dt + \int \sec t \tan t \, dt$

$= \tan t + \sec t + C$

44. $\int_0^{3\pi/2} |\sin x| \, dx = \int_0^{\pi} \sin x \, dx + \int_{\pi}^{3\pi/2} -\sin x \, dx$

$= -\cos x \Big]_0^{\pi} + \cos x \Big]_{\pi}^{3\pi/2}$

$= 1-(-1) + 0-(-1) = 3$

48. 

area is $\int_0^1 y^4 \, dy = y^5/5 \Big]_0^1 = \frac{1}{5}$