Math 1A – Fall 2010 – Haiman

HW 10 Solutions

4.9 2. $F(x) = \frac{1}{6}x^3 - x^2 + 6x + C$

6. Since $f(x) = x^3 - 4x^2 + 4x$, $\quad F(x) = \frac{x^4}{4} - \frac{4x^3}{3} + 2x^2 + C$

14. $F(x) = 3e^x + 7\tan x + C$ (But, caution: this holds on each interval $(k\pi - \pi/2, k\pi + \pi/2)$ in the domain of $f(x)$; the most general antiderivative on the whole domain could have a different C on each such interval.)

26. $f'(x) = 3x^2 - \cos x + A$

$f(x) = x^3 - \sin x + Ax + B$

32. $f(x) = x^2 + x^{-3} + C$; $f(1) = 3 \Rightarrow C = 1$, so $f(x) = x^2 + x^{-3} + 1$.

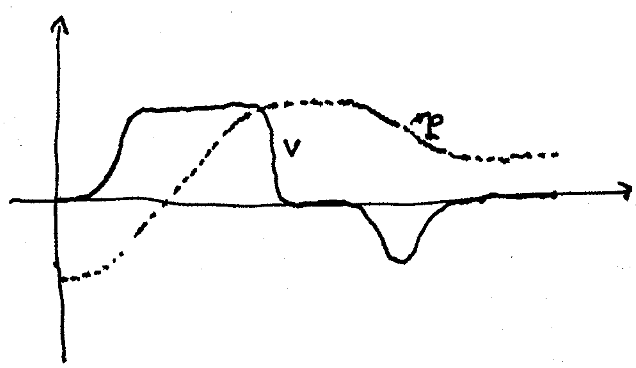
48. $f(x) = \frac{x^4}{4} + C$. This meets the line $y = -x$ when $\frac{x^4}{4} + C = -x$, and we want $f'(x)$ at this point to be -1 for tangency. Thus x at the point of tangency and C should satisfy

$$\frac{x^4}{4} + C = -x, \quad x^3 = -1$$

$$\Rightarrow x = -1, \quad \frac{1}{4} + C = 1, \quad C = 3/4.$$

So the solution is $f(x) = \frac{x^4}{4} + 3/4$; its graph is tangent to $y = -x$ at $(-1, 1)$.

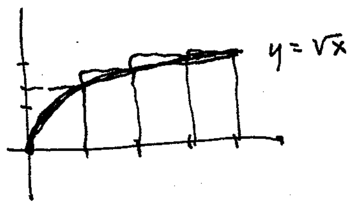
52.



60. Position $s(t)$ satisfies $s''(t) = \cos t + \sin t$, $s(0) = 0$, $s'(0) = 5$

$$s'(t) = \sin t - \cos t + A$$
$$s(t) = -\cos t - \sin t + At + B$$
$$s(0) = -1 + B = 0 \Rightarrow B = 1$$
$$s'(0) = -1 + A = 5 \Rightarrow A = 6$$
$$s(t) = -\cos t - \sin t + 6t + 1$$

5.1 4. a)



$x_i^* = x_i = i$, all $\Delta x_i = 1$

Area estimate is

$$\sum_{i=1}^{4} f(x_i^*)\Delta x_i = \sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4} \approx 6.15$$

b) Now $x_i^* = i-1$, so estimate is $\sqrt{0} + \sqrt{1} + \sqrt{2} + \sqrt{3} \approx 4.15$

Estimate (a) is over and (b) is under, because $\sqrt{x}$ is increasing.

12. a) $12 \cdot 30 + 12 \cdot 28 + 12 \cdot 25 + 12 \cdot 22 + 12 \cdot 24 = 12 \cdot 156 = 1872$ ft.

b) $12(28 + 25 + 22 + 24 + 27) = 12 \cdot 153 = 1836$ ft.

c) There is not enough information to say if these are over or underestimates. If the motorcycle were decelerating steadily, (a) would be over and (b) under, but from the table of velocities we see that it is not decelerating over the whole time interval.

20. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left(5 + \frac{2i}{n}\right)^{10} = \int_5^7 x^{10}\,dx$ is the area under $y = x^{10}$ on the interval $[5,7]$ (or, the area under $y = (5+x)^{10}$ on $[0,2]$).

22. a) $\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n}\left(\frac{i}{n}\right)^3$

b) $= \lim_{n\to\infty} \frac{1}{n^4}\sum_{i=1}^{n} i^3 = \lim_{n\to\infty} \frac{(n(n+1))^2}{4n^4} = \lim_{n\to\infty} \frac{n^4 + 2n^2 + n^2}{4n^4}$

$$= \lim_{n\to\infty}\left(\frac{1}{4} + \frac{1}{2n} + \frac{1}{4n^2}\right) = \frac{1}{4}$$

5.2 8. a) $2(-.6) + 2(.9) + 2(1.8) = 4.2$

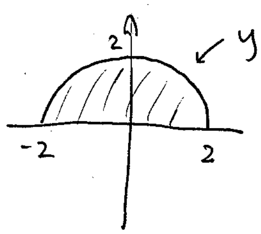
b) $2(-3.4) + 2(-.6) + 2(.9) = -6.2$

c) $2(-2.1) + 2(\cdot 3) + 2(1.4) = -.8$

If ~~the~~ $f(x)$ is increasing, then (a) is over and (b) is under, but we don't have enough information to compare (c) with the true value of $\int_3^9 f(x)\,dx$

18. $\displaystyle\int_{\pi}^{2\pi} \frac{\cos x}{x}\,dx$

36.



$y = \sqrt{4-x^2}$ is a semicircle of radius 2, so

$$\int_{-2}^{2} \sqrt{4-x^2}\,dx = \frac{1}{2}(\pi \cdot 2^2) = 2\pi$$

40. $\displaystyle\int_0^{10} |x-5|\,dx = 2 \cdot \frac{5^2}{2} = 25$