Math 1A - Haiman - Fall 2010

HW 8 Solutions

4.4 6. $\lim_{x\to 2} \frac{x^2+x-6}{x-2} = \lim_{x\to 2} \frac{(x+3)(x-2)}{x-2} = \lim_{x\to 2} x+3 = 5$

No need to use L'Hospital's rule here.

12. $\lim_{t\to 0} \frac{e^{3t}-1}{t} = \lim_{t\to 0} \frac{3e^{3t}}{1} = 3$

(0/0 type)

16. $\lim_{x\to\infty} \frac{x+x^2}{1-2x^2} = \lim_{x\to\infty} \frac{2x+1}{-4x} = \lim_{x\to\infty} \frac{2}{-4} = -\frac{1}{2}$

(∞/∞) (∞/∞ again)

or: $\frac{x+x^2}{1-2x^2} = \frac{1/x+1}{1/x^2-2} \to \frac{0+1}{0-2} = -\frac{1}{2}$

18. $\lim_{x\to\infty} \frac{\ln\ln x}{x} = \lim_{x\to\infty} \frac{\frac{1}{\ln x}\cdot\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{1}{x\ln x} = 0$ since

(∞/∞)

$x\ln x \to \infty$.

Don't use L'Hospital again here, since it's not an indeterminate form!

30. $\lim_{x\to 0} \frac{\cos mx - \cos nx}{x^2} = \lim_{x\to 0} \frac{-m\sin mx + n\sin nx}{2x}$

(0/0) (0/0 again)

$= \lim_{x\to 0} \frac{-m^2\cos mx + n^2\cos nx}{2} = \frac{n^2-m^2}{2}$

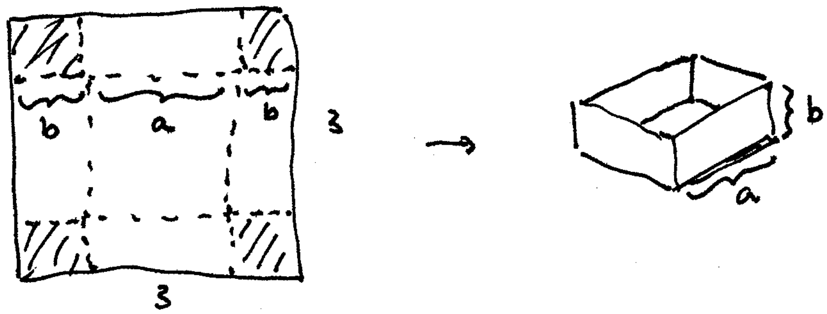
4.7 2. We want to minimize xy when $x-y=100$.

Make everything a function of x: $y = x-100$, $xy = x(x-100) = x^2-100x$

critical number of x^2-100x: $2x-100=0$, $x=50$.

Since $2x-100>0$ for $x>50$, <0 for $x<50$, we have an (absolute) minimum. So the solution is $x=50$, $y=-50$, $xy=-2500$.

20. (a/b)



c) $V = a^2 b$

d) $a + 2b = 3$

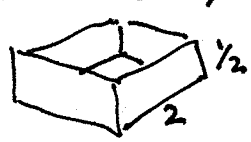
e) $b = \frac{3-a}{2}$, $V = \frac{a^2(3-a)}{2}$

f) To maximize V, $\frac{dV}{da} = \frac{d}{da}\left(\frac{3a^2 - a^3}{2}\right) = \frac{6a - 3a^2}{2}$ has critical #s $a = 0$, $a = 2$. Only values ~~a>0 are relevant~~ $0 \le a \le 3$ are relevant.

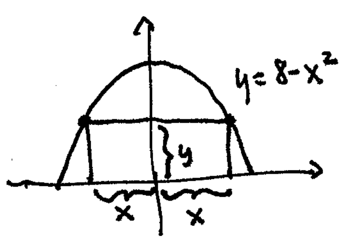
We have

a	V
0	0
2	2
3	0

So V is maximized at $a = 2$, $b = \frac{1}{2}$, $V = 2$ ft^3.



24.



$A = 2xy = 2x(8 - x^2) = 16x - 2x^3$

$\frac{dA}{dx} = 16 - 6x^2 = 0$ at $x = \pm\sqrt{8/3}$.

The relevant range for x is $[0, \sqrt{8}]$, in which $\sqrt{8/3}$ is the only critical number:

x	A
0	0
$\sqrt{8}$	0
$\sqrt{8/3}$	$2\sqrt{8/3}\,\frac{16}{3}$ ← maximum

So the max. area is $\frac{32\sqrt{8/3}}{3} = \frac{64\sqrt{2}}{\sqrt{3}}$.

46. Travel time is $\overline{AB}/2 + \widehat{BC}/4$.

ABC is a right triangle with $\llcorner$ at B, so $\overline{AB} = 4\cos\theta$.

Angle $\alpha = \angle BOC$, where O is the center, is $\alpha = 2\theta$, and

$\widehat{BC} = 2\alpha = 4\theta$.

So $T = 2\cos\theta + \theta$. We want to minimize $T(\theta)$ on $[0, \pi/2]$.

$T' = -2\sin\theta + 1 = 0$ at $\sin\theta = \frac{1}{2}$: $\theta = \pi/6$

θ	T
0	2
$\pi/6$	$\sqrt{3} + \pi/6 \approx 2.26$
$\pi/2$	$\pi/2 \approx 1.57$

So the best plan (corresponding to $\theta = \pi/2$) is to walk along the shore all the way. Note that the critical number in this problem is a max, not a min: it's the <u>worst</u> plan.