Math 1A – Haiman – Fall 2010

HW 6 Solutions

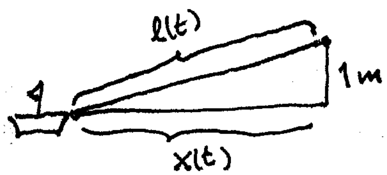
3.9 2. a) $A = \pi r^2 \Rightarrow dA/dt = 2\pi r\, dr/dt$

b) we want dA/dt when $r = 30$ m and $dr/dt = 1$ m/s :

$$dA/dt = 60\pi \text{ m}^2/\text{s}$$

8. $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$. When $y=4$, $x^2+y^2=25 \Rightarrow x=3$. When we also have $\frac{dy}{dt}=6$, $2\cdot 3\frac{dx}{dt} + 2\cdot 4\cdot 6 = 0 \Rightarrow \frac{dx}{dt} = -8$

20.



Given: $dl/dt = -1$ m/s

$l^2 = 1 + x^2$ (from right triangle)

Want $-dx/dt$ when $x = 8$

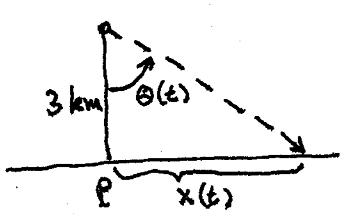
then $l = \sqrt{65}$.

$$2l\, dl/dt = 2x\, dx/dt$$

$$\Rightarrow 2\sqrt{65}\,(-1) = 2\cdot 8\frac{dx}{dt},$$

$$-\frac{dx}{dt} = \frac{8}{\sqrt{65}} \text{ m/s}$$

38.



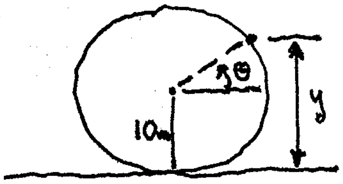
$x = 3\tan\theta$, $d\theta/dt = 8\pi$ /min

$$\frac{dx}{dt} = 3\sec^2\theta\,\frac{d\theta}{dt}.$$

When $x = 1$ km, $\tan\theta = \frac{1}{3}$, $\sec^2\theta = \tan^2\theta + 1 = \frac{10}{9}$, so

$$\frac{dx}{dt} = 3\cdot\frac{10}{9}\cdot 8\pi = \frac{80\pi}{3} \text{ km/min}$$

40.



$y = 10 + 10\sin\theta$ $d\theta/dt = \pi$ /min

$$\frac{dy}{dt} = 10\cos\theta\,\frac{d\theta}{dt}.$$

When $y = 16$ m, $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$ (from $\sin^2\theta + \cos^2\theta = 1$),

so $\frac{dy}{dt} = 10\cdot\frac{4}{5}\cdot\pi = 8\pi$ m/min

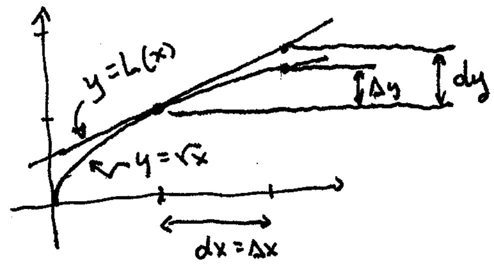
3.10 2. $f(x) = \ln x$, $a = 1$, $f'(x) = \frac{1}{x}$ $f'(a) = \frac{1}{1} = 1$, $f(a) = \ln 1 = 0$

$L(x) = f(a) + f'(a)(x-a) = 0 + 1(x-1) = x-1$

16. a) $dy = \frac{-1}{(x+1)^2}dx$ b) At $x=1$, $dx = -.01$, $dy = \frac{-1}{4}(-.01) = \frac{1}{400} = .0025$

20. $dy = \frac{1}{2\sqrt{x}}dx$. At $x=1$, $dx=1$, $dy = \frac{1}{2}$

For $x=1$, $\Delta x = 1$, $\Delta y = \sqrt{2} - \sqrt{1} = \sqrt{2} - 1 \approx .41$



24. $e^0 = 1$, so $e^{-.015} \approx 1 + dy$ for $x=0$, $dx = -.015$, $y = e^x$.

$dy = e^x dx \mapsto e^0(-.015) = -.015 \Rightarrow e^{-.015} \approx 1 - .015 = .985$

(To 6 places, the exact value is $e^{-.015} \approx .985112$)

28. $\sqrt{100} = 10$, so $\sqrt{99.8} \approx 10 + dy$ for $x = 100$, $dx = -.2$, $y = \sqrt{x}$.

$dy = \frac{1}{2\sqrt{x}}dx \longmapsto \frac{1}{2\sqrt{100}}dx = \frac{-.2}{20} = -.01$, giving $\sqrt{99.8} \approx 10 - .01 = 9.99$

(To 6 places, $\sqrt{99.8} \approx 9.98999$)

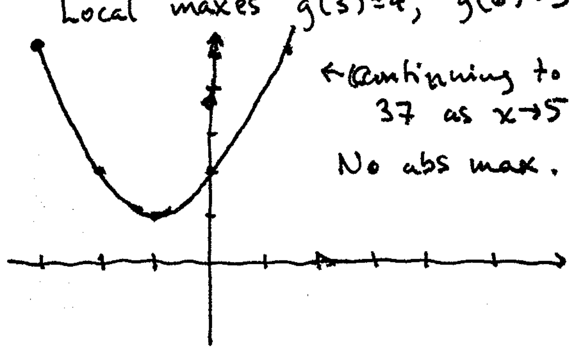
36. The volume of paint needed is the increase in volume from the unpainted dome ($r = 25$ m) to the painted dome ($r = 25 + .0005$ m).

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) \Rightarrow dV = 2\pi r^2 dr$

with $r = 25$ m, $dr = .0005$ m, $dV = .625\pi \approx 1.96$ m^3

4.1 6. No abs. max. Abs. min $g(4) = 1$, which is also a local min. Local maxes $g(3) = 4$, $g(6) = 3$, and another local min $g(2) = 2$.

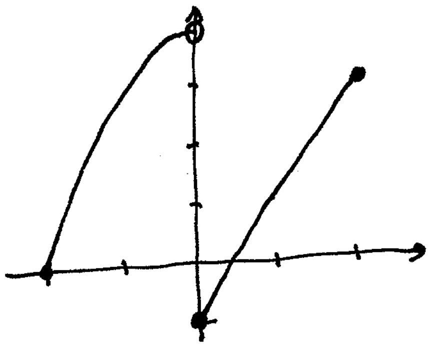
22.



← Continuing to increase, approaching 37 as $x \to 5$

No abs max. Abs. & local min $f(-1) = 1$.

28.



No abs. max.
Abs. + local min at $f(0) = -1$.

34. $g(t) = |3t-4| = \begin{cases} 3t-4 & t \geq 4/3 \\ 4-3t & t \leq 4/3 \end{cases}$

$g'(t) = \begin{cases} 3 & t > 4/3 \\ -3 & t < 4/3 \end{cases}$ $\quad g'(4/3)$ doesn't exist.

So $4/3$ is the only critical number.

48. $f'(x) = 3x^2 - 3 = 3(x^2-1) = 0$ at $x = \pm 1$.
So $f(x)$ has one critical number, $c=1$, in $(0,3)$.
$f(0) = 1$
$f(1) = -1$ ← abs. min.
$f(3) = 19$ ← abs. max.

60. $f'(x) = 1 - \frac{1}{x} = 0$ at $x = 1$:
One crit. number, $c=1$ in $(\frac{1}{2}, 2)$.
$f(1/2) = \frac{1}{2} - \ln\frac{1}{2} = \frac{1}{2} + \ln 2 \approx 1.193$
$f(1) = 1 - 0 = 1$ ← abs. min.
$f(2) = 1 + \ln 2 \approx 1.693$ ← abs. max.

74. $g'(x) = 3(x-5)^2 = 0$ at $x=5$, so 5 is a critical number.
But $y = x^3$ is a strictly increasing function for all x,
hence so are $(x-5)^3$ and $g(x) = 2 + (x-5)^3$. So $g(x)$ has no local extrema.