Math 1A – Haiman – Fall 2010

Hw 5 Solutions

3.4 8. $100(4x-x^2)^{99}(4-2x)$

14. $3\cos^2 x\,(-\sin x)$ [assuming y is meant to be a function of $\underline{x}$, with $\underline{a}$ ~~[illegible]~~ a constant]

16. $-3n\csc^2(n\theta)$ [assuming y function of $\underline{\theta}$, with $\underline{n}$ constant]

22. $-3e^{-5x}\sin 3x - 5e^{-5x}\cos 3x$

42. $\frac{1}{2}\,\frac{1}{\sqrt{x+\sqrt{x+\sqrt{x}}}}\cdot\left(1+\frac{1}{2}\,\frac{1}{\sqrt{x+\sqrt{x}}}\left(1+\frac{1}{2\sqrt{x}}\right)\right)$

64.(a) $F'(2) = f'(f(2))f'(2) = f'(1)f'(2) = 4\cdot 5 = 20$

90. Using product + chain rules on $f(x)\,(g(x)^{-1})'$ gives

$$\left(\frac{f(x)}{g(x)}\right)' = f'(x)\,g(x)^{-1} + f(x)\left(-g(x)^{-2}g'(x)\right)$$

$$= \frac{f'(x)g(x)}{g(x)^2} - \frac{f(x)g'(x)}{g(x)^2} = \frac{f'(x)\,g(x) - f(x)g'(x)}{g(x)^2}$$

3.5 4. a) $\cos x + \sqrt{y} = 5 \Rightarrow -\sin x + \frac{1}{2\sqrt{y}}\,y' = 0$

$$\Rightarrow y' = 2\sqrt{y}\,\sin x$$

b) $y = (5-\cos x)^2 \quad y' = 2(5-\cos x)(\sin x)$

c) putting $y = (5-\cos x)^2$ in (a) gives $y' = 2\sqrt{(5-\cos x)^2}\,\sin x$
$= 2(5-\cos x)\sin x$

[Note that $5-\cos x > 0$, so $\sqrt{(5-\cos x)^2} = |5-\cos x| = 5-\cos x$]

16. $\cancel{\frac{1}{2\sqrt{x+y}}\,y' = 2xy^2 + 2x^2y\,y' \Rightarrow y' = \frac{2xy^2}{\frac{1}{2\sqrt{x+y}} - 2x^2y}}$

$$\frac{1}{2\sqrt{x+y}}(1+y') = 2xy^2 + 2x^2y\,y'$$

$$\Rightarrow y' = \frac{2xy^2 - 1/(2\sqrt{x+y})}{1/(2\sqrt{x+y}) - 2x^2y}$$

$$= \frac{1-4xy^2\sqrt{x+y}}{4x^2y\sqrt{x+y}-1}$$

$$= \frac{1-4x^3y^4}{4x^4y^3-1}$$

22. $g'(x) + \sin g(x) + x(\cos g(x))\,g'(x) = 2x$.

At $x=0$, $g(x) + x\sin g(x) = x^2 \Rightarrow g(0) = 0$, so

$g'(0) + \sin 0 + 0 = 0 \Rightarrow g'(0) = 0$.

36. $x^4+y^4=a^4 \Rightarrow 4x^3+4y^3y'=0 \Rightarrow y'=-x^3/y^3$

Then $y'' = \dfrac{-3x^2y^3+3x^3y^2y'}{y^6} = \dfrac{-3x^2y^3-3x^6/y}{y^6} = -3\left(\dfrac{x^2}{y^3}+\dfrac{x^6}{y^7}\right)$

54. $y' = \dfrac{1}{1+\left(\sqrt{\frac{1-x}{1+x}}\right)^2} \dfrac{1}{2\sqrt{\frac{1-x}{1+x}}} \dfrac{-2}{(1+x)^2}$

$= \dfrac{-1}{\left(1+\frac{1-x}{1+x}\right)(1+x)^2\sqrt{\frac{1-x}{1+x}}} = \dfrac{-1}{2(1+x)\sqrt{\frac{1-x}{1+x}}} = \dfrac{-1}{2\sqrt{1-x^2}}$

3.6 6. $f'(x) = \dfrac{1}{\ln 5}\dfrac{1}{xe^x}(e^x+xe^x) = \dfrac{x+1}{(\ln 5)x}$

16. $y' = -\dfrac{1}{(\ln x)^2}\dfrac{1}{x} = \dfrac{-1}{x(\ln x)^2}$

24. $y' = \dfrac{1-2\ln x}{x^3}$

38. $\ln y = \frac{1}{2}\ln x + x^2 + 10\ln(x^2+1)$

$y'/y = (\ln y)' = \dfrac{1}{2x} + 2x + \dfrac{10}{x^2+1}\cdot 2x$

$y' = \left(\dfrac{1}{2x}+2x+\dfrac{20x}{x^2+1}\right)\sqrt{x}\,e^{x^2}(x^2+1)^{10}$

46. $\ln y = \ln x \ \ln\sin x$

$y'/y = (\ln y)' = \dfrac{1}{x}\ln\sin x + \ln x \dfrac{1}{\sin x}\cos x$

$y' = \left(\dfrac{\ln\sin x}{x} + \ln x \cot x\right)(\sin x)^{\ln x}$