Math 1A - Hakman - Fall 2010

HW 4 Solutions

**3.2** 2. Quotient Rule gives $F'(x) = \dfrac{\left(1-\frac{9}{2}\sqrt{x}\right)\sqrt{x} - (x - 3x\sqrt{x})\cdot \frac{1}{2}\frac{1}{\sqrt{x}}}{x}$

Simplifying first:

$F(x) = \sqrt{x} - 3x \qquad F'(x) = \frac{1}{2}\frac{1}{\sqrt{x}} - 3$

4. $g'(x) = \left(\frac{1}{2}x^{-1/2} + x^{1/2}\right)e^x = \left(\frac{1}{2\sqrt{x}} + \sqrt{x}\right)e^x$

8. $f'(t) = \dfrac{(4+t^2)\cdot 2 - 2t(2t)}{(4+t^2)^2} = \dfrac{8-2t^2}{(4+t^2)^2}$

10. $Y'(u) = (-2u^{-3} - 3u^{-4})(u^5 - 2u^2) + (u^{-2} + u^{-3})(5u^4 - 4u)$

18. $y'(s) = \dfrac{-(1+ke^s)}{(s+ke^s)^2}$

28. $f'(x) = \left(\frac{5}{2}x^{3/2} + x^{5/2}\right)e^x \qquad f''(x) = \left(\frac{15}{4}x^{1/2} + 5x^{3/2} + x^{5/2}\right)e^x$

48. a) From the graph we get $F(2) = 3 \quad F'(2) = 0 \quad G(2) = 2 \quad G'(2) = \frac{1}{2}$.

So $P'(2) = F'(2)G(2) + F(2)G'(2) = 0 + \frac{3}{2} = \frac{3}{2}$

b) $F(7) = 5, \quad F'(7) = \frac{1}{4}, \quad G(7) = 1, \quad G'(7) = -\frac{2}{3}$

$\Rightarrow Q'(7) = \dfrac{F'(7)G(7) - F(7)G'(7)}{G(7)^2} = \dfrac{\frac{1}{4} + \frac{10}{3}}{1} = \dfrac{43}{12}$

52. We need points where the slope is $\frac{1}{2}$.

$y' = \dfrac{(x+1)\cdot 1 - (x-1)\cdot 1}{(x+1)^2} = \dfrac{2}{(x+1)^2}$ so $y'(x) = \frac{1}{2}$ at $(x+1)^2 = 4$

$x+1 = \pm 2$

$x \in \{1, -3\}$

Tangent line at $(1,0)$: $y = \frac{1}{2}(x-1)$

at $(-3,2)$: $y - 2 = \frac{1}{2}(x+3)$

or $y = \frac{1}{2}(x+7)$

58. a) Take $f(x)=1$ (so $f'(x)=0$) in $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

to get $\frac{-g'(x)}{g(x)^2}$

b) See Ex. 18

c) $\frac{d}{dx}(x^{-n}) = \frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{-n x^{n-1}}{x^{2n}}$ by Reciprocal Rule

$= \frac{-n}{x^{n+1}} = -n x^{-n-1}$ ☺

3.3 2. $f'(x) = \frac{1}{2}x^{-1/2}\sin x + \sqrt{x}\cos x$

6. $g'(t) = 4\sec t \tan t + \sec^2 t$

12. Simplify first: $y = \cot x - \csc x$

$y' = -\csc^2 x - \csc x \cot x$

16. $y' = 2x\sin x \tan x + x^2 \cos x \tan x + x^2 \sin x \sec^2 x$

$= 2x \sin x \tan x + x^2 \sin x + x^2 \sec x \tan x$

18. $\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right) = \frac{-\cos' x}{\cos^2 x} = \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x}\frac{\sin x}{\cos x}$

$= \sec x \tan x$

34. $y' = \frac{-(2+\sin x)\sin x - (\cos x)(\cos x)}{(2+\sin x)^2}$

$= \frac{-2\sin x - (\sin^2 x + \cos^2 x)}{(2+\sin x)^2} = \frac{-2\sin x - 1}{(2+\sin x)^2}$.

$y'(x) = 0$ when $\sin x = -1/2$ : $x = -\frac{\pi}{6} + 2k\pi$

or $x = -\frac{5\pi}{6} + 2k\pi$, $k$ any integer

42. $\lim_{\theta\to 0} \frac{\cos\theta - 1}{\sin\theta} = \lim_{\theta\to 0} \underbrace{\frac{\cos\theta - 1}{\theta}}_{\to 0} \underbrace{\frac{\theta}{\sin\theta}}_{\to 1} = 0.$

44. $\lim_{t\to 0} \frac{\sin^2 3t}{t^2} = \lim_{t\to 0} 9\left(\frac{\sin 3t}{3t}\right)^2 = 9$