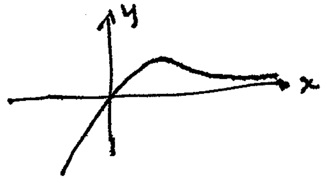
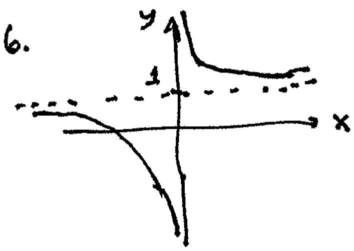
Math 1A – Haiman – Fall 2010
Homework 3 Solutions

2.6 2(a). Can ~~must~~ intersect a vertical asymptote, but not cross one.
Can intersect a horizontal asymptote, e.g.
$y = xe^{-x}$ has $\lim_{x\to\infty} xe^{-x} = 0$, so the x-axis is a horizontal asymptote, and the graph crosses it at the origin



6.



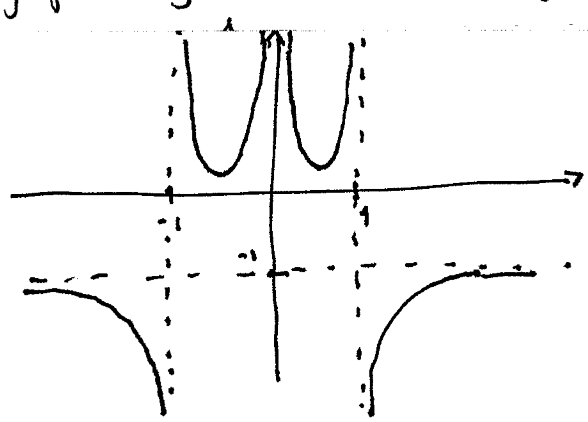
18. $$\lim_{y\to\infty} \frac{2-3y^2}{5y^2+4y} = \lim_{y\to\infty} \frac{2/y^2-3}{5+4/y} = -\frac{3}{5}$$

26. $$\lim_{x\to-\infty} \left(x + \sqrt{x^2+2x}\right) = \lim_{x\to-\infty} \frac{-2x}{x-\sqrt{x^2+2x}} = \lim_{x\to-\infty} \frac{-2}{1+\sqrt{1+2/x}} = -1$$

(note that $\frac{\sqrt{x^2+2x}}{x} = -\sqrt{1+2/x}$ for $x<0$)

30. $\lim_{x\to\infty} \sqrt{x^2+1} = +\infty$

42. Horizontal asymptote $y = -1$. Vertical asymptotes $x = 0, \pm 1$



66. a) $x > 10^8$

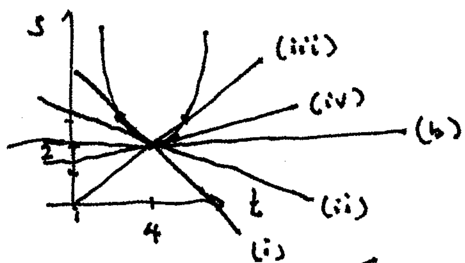
b) We must prove that ~~for~~ every $\varepsilon > 0$ there is an N such that $x > N$ implies $|1/\sqrt{x}| < \varepsilon$. I claim $N = \frac{1}{\varepsilon^2}$ works: if $x > \frac{1}{\varepsilon^2}$, then (since both sides are >0) $\frac{1}{x} < \varepsilon^2$, so $\frac{1}{\sqrt{x}} < \varepsilon$.
Since $\frac{1}{\sqrt{x}} > 0$, $\left|\frac{1}{\sqrt{x}}\right| = \frac{1}{\sqrt{x}} < \varepsilon$.

<u>2.7</u> 6. $y' = 6x^2 - 5$, so $y'(-1) = 1$. The line is $y - 3 = 1(x - (-1))$, or $y = x + 4$.

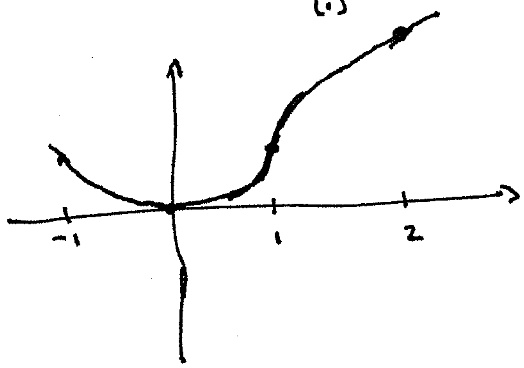
16. a) (i) -1, (ii) $-.5$ (iii) 1 (iv) $.5$

b) $s'(t) = 2t - 8 \mapsto 0$ at $t = 4$

c)



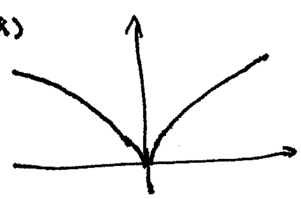
20.



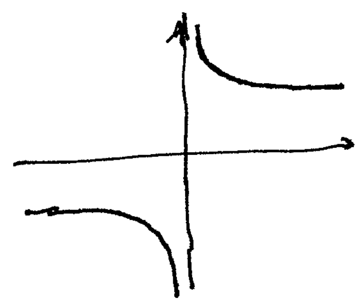
30. $f(x) = \sqrt{3x+1}$

$f'(a) = \dfrac{3}{2\sqrt{3x+1}}$

<u>2.8</u> 8. $y = f(x)$



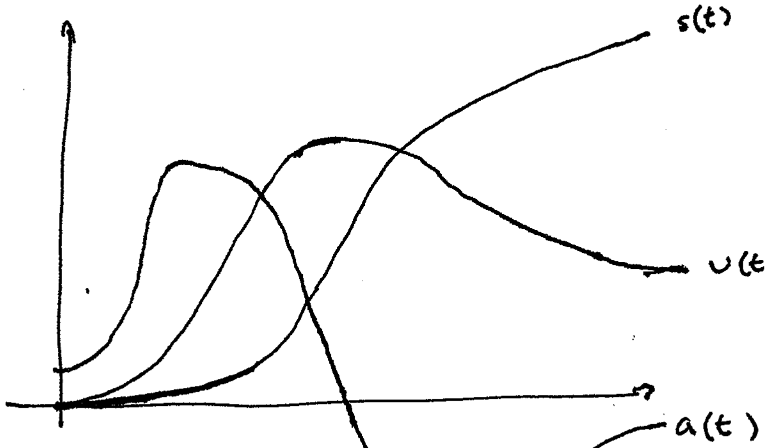
$y = f'(x)$



($f(x)$ is not differentiable at 0)

24. $(x + \sqrt{x})' = 1 + \dfrac{1}{2\sqrt{x}}$. Domain of $f(x)$ is $[0, \infty)$; of $f'(x)$ is $(0, \infty)$.

48.



3.1 8. $f'(t) = 3t^5 - 12t^3 + 1$

10. $h'(x) = 2x+3 + 2(x-2) = 4x-1$

24. $y' = 1 + x^{-3/2}$

32. $y' = e^{x+1}$ (note that $e^{x+1} = e \cdot e^x$)

34. $y' = 4x^3 + 4x - 1 \Rightarrow$ slope is $y'(1) = 7$
tangent line is $(y-2) = 7(x-1)$, or $y = 7x-5$

54. Slope of tangent line at $(a, a\sqrt{a})$ is $y'(a) = \frac{3}{2}\sqrt{a}$. To be parallel to $y = 1+3x$, want slope $= 3$, so $a = 4$, $y(a) = 8$, and the line with slope 3 through $(4, 8)$ is $y - 8 = 3(x-4)$, or $y = 3x - 4$.

74. $y = \frac{3}{2}x + 6$ has slope $\frac{3}{2}$.

The curve $y = c\sqrt{x}$ meets the line $y = \frac{3}{2}x + 6$ at the point where $\frac{3}{2}x + 6 = c\sqrt{x}$, and its slope is $\frac{3}{2}$ where $y' = \frac{1}{2}\frac{c}{\sqrt{x}} = \frac{3}{2}$. So we need $c = 3\sqrt{x}$ and $\frac{3}{2}x + 6 = c\sqrt{x}$ to hold for the same x. Then $c\sqrt{x} = 3\sqrt{x}\sqrt{x} = 3x$, so $\frac{3}{2}x + 6 = 3x$, $6 = \frac{3}{2}x$, $x = 4$, $c = 6$.

Check: with $c = 6$, at $x=4$ we have $\frac{3}{2}x + 6 = 12 = 6\sqrt{x}$, so the curve $y = 6\sqrt{x}$ and the line $\frac{3}{2}x + 6$ meet at $x = 4$. The slope of the curve is $y'(4) = \frac{3}{\sqrt{4}} = \frac{3}{2}$, same as the line, so the line and the curve are tangent.