Math 1A - Lect. 3 - Haiman - Fall 2010
HW 2 Solutions

2.3

26. $\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right)=\lim_{t\to 0}\frac{1}{1+t}=1$

38. Since $-1\le \sin(\pi/x)\le 1$ for all $x>0$, we have
$e^{-1}\le e^{\sin(\pi/x)}\le e$, so $e^{-1}\sqrt{x}\le \sqrt{x}\,e^{\sin(\pi/x)}\le e\sqrt{x}$.
By Squeeze Theorem, since $\lim_{x\to 0} e^{-1}\sqrt{x}=0=\lim_{x\to 0} e\sqrt{x}$,
we get $\lim_{x\to 0}\sqrt{x}\,e^{\sin(\pi/x)}=0$.

56 (b) $\lim_{x\to 0}\frac{f(x)}{x}=\lim_{x\to 0} x\,\frac{f(x)}{x^2}=\left(\lim_{x\to 0} x\right)\left(\lim_{x\to 0}\frac{f(x)}{x^2}\right)=0\cdot 5=0$.

60. $\lim_{x\to 2}\frac{\sqrt{6-x}-2}{\sqrt{3-x}-1}=\lim_{x\to 2}\frac{((6-x)-2^2)(\sqrt{3-x}+1)}{((3-x)-1^2)(\sqrt{6-x}+2)}=\lim_{x\to 2}\frac{2-x}{2-x}\,\frac{\sqrt{3-x}+1}{\sqrt{6-x}+2}$

$=\lim_{x\to 2}\frac{\sqrt{3-x}+1}{\sqrt{6-x}+2}=\frac{\sqrt{3-2}+1}{\sqrt{6-2}+2}=\frac{2}{4}=\frac{1}{2}$

2.4

36. Given $\varepsilon>0$, assume $\varepsilon<\frac{1}{2}$ (if $\varepsilon\ge\frac{1}{2}$, then a δ that works for, say, $\varepsilon=1/3$ will also work for $\varepsilon>1/3$), and define $\delta=\min\left(\frac{1}{\frac{1}{2}-\varepsilon}-2,\ 2-\frac{1}{\frac{1}{2}+\varepsilon}\right)$.

Notice that $\frac{1}{\frac{1}{2}-\varepsilon}>\frac{1}{1/2}=2$ and $\frac{1}{\frac{1}{2}+\varepsilon}<\frac{1}{1/2}=2$, so $\delta>0$.

If $x\in(2-\delta,2+\delta)$, then

$x>2-\delta \Rightarrow x>2-\left(2-\frac{1}{\frac{1}{2}+\varepsilon}\right)=\frac{1}{\frac{1}{2}+\varepsilon}\Rightarrow \frac{1}{x}<\frac{1}{2}+\varepsilon$

$x<2+\delta \Rightarrow x<2+\left(\frac{1}{\frac{1}{2}-\varepsilon}-2\right)=\frac{1}{\frac{1}{2}-\varepsilon}\Rightarrow \frac{1}{x}>\frac{1}{2}-\varepsilon$.

So $\frac{1}{x}\in\left(\frac{1}{2}-\varepsilon,\frac{1}{2}+\varepsilon\right)$, i.e. $\left|\frac{1}{x}-\frac{1}{2}\right|<\varepsilon$.

Since for any $\varepsilon>0$ we have found a $\delta>0$ such that $x\in(2-\delta,2+\delta)\Rightarrow\left|\frac{1}{x}-\frac{1}{2}\right|<\varepsilon$, we proved $\lim_{x\to 2}\frac{1}{x}=\frac{1}{2}$.

42. Given N (we can assume $N>0$), let $\delta = \frac{1}{\sqrt[4]{N}}$.

If $0 < |x+3| < \delta$ then $(x+3)^4 = |x+3|^4 < \delta^4 = \frac{1}{N}$,

So $\frac{1}{(x+3)^4} > N$. (Note $\frac{1}{(x+3)^4}$ is defined since $x \neq -3$).

This proves $\lim_{x\to -3} \frac{1}{(x+3)^4} = +\infty$.

2.5

10. By limit laws we calculate

$$\lim_{x\to 4} x^2 + \sqrt{7-x} = \lim_{x\to 4} x^2 + \lim_{x\to 4} \sqrt{7-x}$$
$$= \left(\lim_{x\to 4} x\right)^2 + \sqrt{\lim_{x\to 4}(7-x)} = \left(\lim_{x\to 4} x\right)^2 + \sqrt{7 - \lim_{x\to 4} x}$$
$$= 4^2 + \sqrt{7-4}.$$

This shows $f(x) = x^2 + \sqrt{7-x}$ has $\lim_{x\to 4} f(x) = f(4)$, so f is continuous at 4.

24. Domain of $\frac{\sin x}{x+1}$ is all $x \neq -1$. Since $\sin x$, x, and 1 are continuous, so are $x+1$, and $\frac{\sin x}{x+1}$ at all x where $x+1 \neq 0$.

36. $f(x) = \begin{cases} \sin x & x < \pi/4 \\ \cos x & x \geq \pi/4 \end{cases}$ is continuous on $(\pi/4, \infty)$ and $(-\infty, \pi/4)$

since $\sin x$ and $\cos x$ are continuous. At $\pi/4$ we have

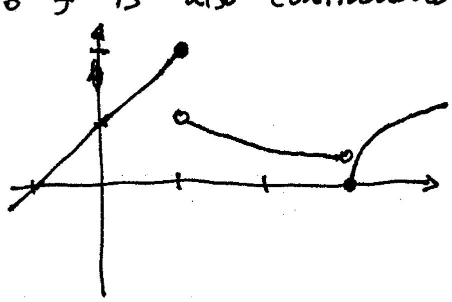
$f(\pi/4) = \cos(\pi/4) = \lim_{x\to \pi/4^+} \cos x = \lim_{x\to\pi/4^+} f(x)$

and $\lim_{x\to\pi/4^-} f(x) = \lim_{x\to\pi/4^-} \sin x = \sin(\pi/4)$.

Since $\cos(\pi/4) = \sin(\pi/4) = \frac{\sqrt{2}}{2}$, we see that $\lim_{x\to\pi/4} f(x) = f(\pi/4) = \sqrt{2}/2$,

so f is also continuous at $\pi/4$.

38.



$f(x) = \begin{cases} x+1 & x \leq 1 \\ 1/x & 1 < x < 3 \\ \sqrt{x-3} & x \geq 3 \end{cases}$

is discontinuous at 1, 3; continuous from left at 1, from right at 3.

50. Let $f(x) = \ln x - e^{-x}$. Then $f(1) = -e^{-1} < 0$, $f(2) = \ln 2 - e^{-2} \approx .558\ldots > 0$

By IVT, since $f(x)$ is continuous, there must be a $c \in (1,2)$ such that $f(c) = 0$, i.e. $\ln c = e^{-c}$.