Prof. Haiman | Math 1A—Calculus | Fall, 2006

Quiz 9 solutions—version B

Name ______________________ Student ID Number ______________________

1. Find the limit, if it exists as a number or as an infinite limit.

$$\lim_{x\to 0} \frac{e^x - e^{-x}}{x}$$

This has the "0/0" form, so use L'Hospital's rule:

$$\lim_{x\to 0} \frac{e^x - e^{-x}}{x} = \lim_{x\to 0} \frac{2e^x}{1} = 2.$$

2. Find the limit, if it exists as a number or as an infinite limit.

$$\lim_{x\to \pi/2} (\sec x - \tan x)$$

This has the "$\infty - \infty$" form. Rewrite it as

$$\lim_{x\to \pi/2} \frac{1-\sin x}{\cos x},$$

which has the "0/0" form. Then L'Hospital's rule gives

$$\lim_{x\to \pi/2} \frac{1-\sin x}{\cos x} = \lim_{x\to \pi/2} \frac{-\cos x}{-\sin x} = 0.$$

3. Sketch a graph of the function, showing any horizontal, vertical or slant asymptotes:

$$y = \frac{x^3+2}{x^2-4}$$

The denominator vanishes at $x = \pm 2$ while the numerator does not, giving vertical asymptotes at $x = \pm 2$. The form of the fraction suggests a slant asymptote with slope 1. To determine it precisely, compute

$$\lim_{x\to\infty} \frac{x^3+2}{x^2-4} - x = \lim_{x\to\infty} \frac{4x+2}{x^2-4} = 0,$$

with the same limit as $x \to -\infty$. This shows that the line $y = x$ is a slant asymptote in both directions.