Prof. Haiman | Math 1A—Calculus | Fall, 2006

Quiz 10 solutions—version A

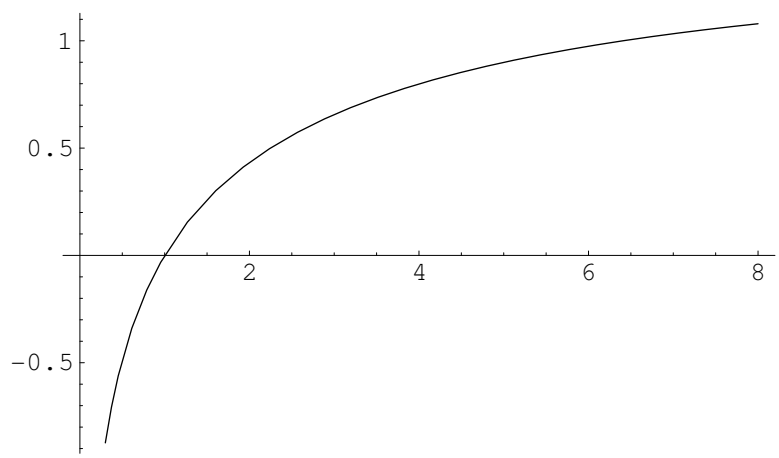
Name _____ Student ID Number _____

1. Use two steps of Newton's method, starting with $x_1 = 1$, to approximate the solution of the equation $x^3 = x + 1$. Express answers as fractions p/q where p and q are integers.

Take $f(x) = x^3 - x - 1$, so $f'(x) = 3x^2 - 1$. First step: $f(x_1) = -1$, $f'(x_1) = 2$ gives $x_2 = 1 - (-1)/2 = 3/2$. Second step: $f(x_2) = 7/8$, $f'(x_2) = 23/4$ gives $x_3 = 3/2 - (7/8)/(23/4) = 31/23$.

For comparison, $31/23 \approx 1.3478$. The true solution to five decimal places is 1.32472, which you would get after two more steps of Newton's method.

2. A computer plotted graph of the function $f(x) = \ln(x) - x^{1/3} + 1$ is shown below. In what way does the behavior of $f(x)$ for large x differ from what the graph suggests? Justify your answer.



From the graph it looks like $f(x)$ is an increasing function. However, since $x^{1/3}$ is asymptotically large compared to $\ln(x)+1$, $f(x)$ is actually decreasing for large x, and $f(x) \to -\infty$ as $x \to \infty$. This plot of $f(x)$ on a different scale gives a better picture.