Math 1A Calculus Fall, 2004

Prof. Haiman

Practice Exam for Midterm 1

You may bring one sheet of notes to the exam (on ordinary sized paper, written on both sides). No other notes, books, calculators or computers may be used.

Each question is worth the same number of points. The practice exam has more questions than will be on the real exam.

Show work or briefly indicate a reason for each of your answers. For most questions, formulas showing one or two steps in your calculations, or a few words of explanation, will be sufficient. You can take the accompanying solutions to this practice exam as a guide to what is expected. Partial credit for incomplete or wrong answers will usually not be given.

1. Find the domain and range of the function

$$f(x) = \frac{1}{(x-2)^2}.$$

2. Express the function

$$u(t) = \frac{\cos t}{1+\cos t}$$

as a composite $f \circ g$ of two other functions.

3. An exponential function $f(x) = Ca^x$ has $f(1) = 10$ and $f(3) = 40$. Find the constants C and a.

4. Evaluate the limit, if it exists (possibly as an infinite limit).

$$\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3}.$$

5. Evaluate the limit, if it exists (possibly as an infinite limit).

$$\lim_{x\to \pi/2} \tan x$$

6. Evaluate the limit, if it exists (possibly as an infinite limit).

$$\lim_{x\to \infty} \tan^{-1}(x^2 - x^4)$$

7. In the definition of the limit

$$\lim_{x\to 1}(5-3x) = 2,$$

find a value of δ that works for $\varepsilon = 0.1$.

8. Prove that there is at least one real solution of the equation $xe^x = 1$.

9. For what value of the constant c is the function

$$f(x) = \begin{cases} x + c & \text{if } x \leq -1 \\ x^2 - c & \text{if } x > -1 \end{cases}$$

continuous on $(-\infty, \infty)$?

10. Differentiate the function

$$f(x) = (\sqrt{x} - 1)(e^x + x)$$

11. Differentiate the function

$$f(x) = \frac{1}{x^3 - x + 2}$$

12. Find an equation of the tangent line to the curve

$$y = \sqrt{x}$$

at the point $(4, 2)$.