Math 1A | Calculus | Fall, 2004
Prof. Haiman

Practice Final Exam

1. Differentiate the function
$$y = \frac{(x+1)\sqrt{x+2}}{\sqrt[3]{x+3}}.$$

2. Evaluate the limit if it exists (possibly as an infinite limit).
$$\text{(a)}\ \lim_{x\to 1^+} \frac{x}{1-x} \qquad \text{(b)}\ \lim_{x\to 1^-} \frac{x}{1-x} \qquad \text{(c)}\ \lim_{x\to 1} \frac{x}{1-x}$$

3. Find all points P on the curve $y = x^2 + 1$ with the property that the tangent line at P passes through the origin.

4. Use a linear approximation to estimate $\sqrt{37}$.

5. If $\sin(y - x) = y + x$, express dy/dx in terms of x and y.

6. Find the constant a for which $f(x) = x^3 + ax^2$ has an inflection point at $x = 1$. For this value of a, find the intervals of concavity of $f(x)$.

7. Use Newton's method to find the root of $x^4 + x - 4 = 0$ in the interval $[1, 2]$, correct to 6 decimal places.

8. Find the points on the parabola $y = x^2$ closest to $(0, 1)$.

9. Find the limit.
$$\lim_{x\to 1} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$$

10. Evaluate the integral.
$$\int_1^2 x\sqrt{x-1}\, dx$$

11. Find the area enclosed by the lines $x = 0$, $y = 1$ and the curve $y = \sqrt[3]{x}$.

12. Evaluate the integral.
$$\int_0^{\pi/2} \left| \cos x - \frac{1}{2} \right|\, dx.$$

13. Differentiate the function

$$f(x) = \int_x^{2x} \frac{e^t}{t}\, dt.$$

14. Find the most general function $f(x)$ for which $f''(x) = \cos x$.

15. Find an interval $[0, c]$ on which the average value of the function $f(x) = x^2 + 2$ is equal to 5.

16. Set up an integral for the volume of the solid obtained by rotating the region enclosed by the x axis, the line $x = 2$, and the curve $y = \ln x$ about the y axis, using
(a) the method of slices;
(b) the method of cylindrical shells.
Evaluate one of these integrals to find the volume.

17. Find the volume of a pyramid with a square base of length 2 on each side, and height 3.

18. Evaluate the limit by expressing it as an integral.

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \frac{i^2}{n^2}.$$