Calculus Prof. Haiman

Quiz 9 Solution (Version A)

1. Find the most general antiderivative of the function

$$f(x) = \frac{4 + 3x - x^5}{x^2}.$$

$$f(x) = 4x^{-2} + 3x^{-1} - x^{3}$$

$$F(x) = -4x^{-1} + 3\ln x - \frac{x^{4}}{4} + C.$$

2. Using Newton's method to approximate a solution to $x^2 - 3x + 1 = 0$, with an initial guess $x_1 = 0$, find the next two approximants x_2 and x_3 (express your answers as exact fractions). Check by verifying that $x_3^2 - 3x_3 + 1$ is close to zero.

Newton's formula gives

$$x_{n+1} = x_n - \frac{x_n^2 - 3x_n + 1}{2x_n - 3}.$$

From this, compute $x_2 = 1/3$, $x_3 = 8/21$. To check, compute $x_3^2 - 3x_3 + 1 = 1/441$, quite a small number.