

**Quiz 8 Solution (Version B)**

1. Find the limit

$$\lim_{x \rightarrow \infty} x \ln(1 + 1/x)$$

$$\lim_{x \rightarrow \infty} x \ln(1 + 1/x) = \lim_{x \rightarrow \infty} \frac{\ln(1 + 1/x)}{1/x}.$$

Now we have a 0/0 type limit and can apply L'Hospital's rule to get

$$\lim_{x \rightarrow \infty} \frac{-x^{-2}/(1 + 1/x)}{-x^{-2}} = 1.$$

2. Find the largest possible volume for a circular cylinder of height  $h$  and radius  $r$ , if it is required that  $h + 2r = 1$  m.

The volume is

$$V = \pi r^2 h = \pi r^2 (1 - 2r) = \pi(r^2 - 2r^3).$$

We are to maximize this on the interval  $0 \leq r \leq 1/2$ .

$$dV/dt = \pi(2r - 6r^2) = 2\pi r(1 - 3r),$$

giving a critical point at  $r = 1/3$ , as well as the endpoints  $r = 0, 1/2$ . We have  $V = 0$  at the endpoints, so the absolute maximum is  $V = \pi/27 \text{ m}^3$ , occurring when  $r = h = 1/3$ .