Math 1A | Calculus | Fall, 2004

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Quiz 8 Solution (Version A)

1. Find the limit
$$\lim_{x\to 1} (\ln x)(\tan \pi x/2)$$

$$\lim_{x\to 1} (\ln x)(\tan \pi x/2) = \lim_{x\to 1} \frac{\ln x}{\cot \pi x/2}.$$

Now we have a 0/0 type limit and can apply L'Hospital's rule to get
$$\lim_{x\to 1} \frac{1/x}{-(\pi/2)\csc^2 \pi x/2} = -2/\pi.$$

2. A rectangular box has height h, width w and depth d. Find the largest possible volume for the box if it is required that $w = 2h$, and the total perimeter $h + w + d$ is 3 m.

The constraints imply $3h + d = 3$, so $d = 3 - 3h$. The volume is
$$V = hwd = h(2h)(3 - 3h) = 6h^2 - 6h^3.$$

We are to maximize this on the interval $0 \le h \le 1$.
$$dV/dt = 12h - 18h^2 = 6h(2 - 3h)$$

giving a critical point at $h = 2/3$, in addition to the endpoints $h = 0, 1$ of the domain. We have $V = 0$ at the endpoints, so the absolute maximum is $V = (2/3)(4/3)(1) = 8/9\,\mathrm{m}^3$, with $h = 2/3$, $w = 4/3$, $d = 1$.