Quiz 7 Solution (Version B)

Find (a) the local and absolute minima and maxima, (b) the intervals of increase and decrease, (c) the intervals of concavity, and (d) the inflection points of the function

$$f(x) = \sqrt{x} - \sqrt[3]{x}.$$

We **do** count local minima or maxima at endpoints of the domain of the function, if there are any.

The domain of f is $[0, \infty)$. The first derivative is $f'(x) = (1/2)x^{-1/2} - (1/3)x^{-2/3} = (1/2)x^{-2/3}(x^{1/6} - 2/3)$, giving a critical point at $x = (2/3)^6$. The function f is decreasing on $[0, (2/3)^6)$ and increasing on $((2/3)^6, \infty)$. It has a local but not absolute maximum at $x = 0$, $f(0) = 0$, and an absolute minimum at $x = (2/3)^6$, $f((2/3)^6) = (2/3)^3 - (2/3)^2$.

The second derivative is $f''(x) = -(1/4)x^{-3/2} + (2/9)x^{-5/3} = -(1/4)x^{-5/3}(x^{1/6} - 8/9)$. It has an inflection point at $x = (8/9)^6$, is concave upwards on $(0, (8/9)^6)$ and is concave downwards on $((8/9)^6, \infty)$.