

Quiz 7 Solution (Version A)

Find (a) the local and absolute minima and maxima, (b) the intervals of increase and decrease, (c) the intervals of concavity, and (d) the inflection points of the function

$$f(x) = x^4 - 2x^2.$$

The first derivative is $f'(x) = 4x^3 - 4x = 4x(x+1)(x-1)$, so the critical points are $x = 0, \pm 1$. The function f is increasing on $(-1, 0)$ and $(1, \infty)$, decreasing on $(-\infty, -1)$ and $(0, 1)$. It has absolute minima at $x = \pm 1$, $f(\pm 1) = -1$, and a local but not absolute maximum at $x = 0$, $f(0) = 0$.

The second derivative is $f''(x) = 12x^2 - 4 = 12(x + 1/\sqrt{3})(x - 1/\sqrt{3})$. The function is concave upwards on $(-\infty, -1/\sqrt{3})$ and $(1/\sqrt{3}, \infty)$, concave downwards on $(-1/\sqrt{3}, 1/\sqrt{3})$. It has inflection points at $x = \pm 1/\sqrt{3}$.