Quiz 12 Solution (Version B)

1. Evaluate the indefinite integral

$$\int \frac{\sin x}{1 + \cos x}\, dx.$$
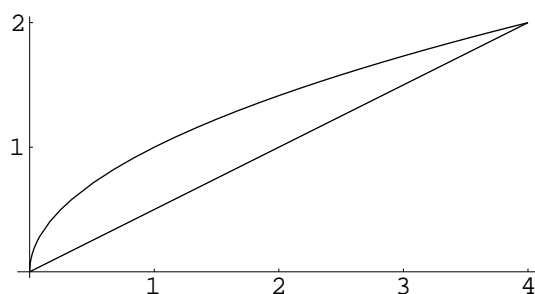
Substitute $u = 1 + \cos x$, $du = -\sin x\, dx$:

$$\int \frac{\sin x}{1 + \cos x}\, dx = \int -\frac{du}{u}$$
$$= -\ln u + C$$
$$= -\ln(1 + \cos x) + C.$$

2. Sketch the region enclosed by the curves $y = \sqrt{x}$, $y = x/2$, and find its area.

Find the endpoints by solving

$$\sqrt{x} = x/2 \;\Rightarrow\; x = 2\sqrt{x} \;\Rightarrow\; x^2 = 4x \;\Rightarrow\; x(x - 4) = 0 \;\Rightarrow\; x = 0 \text{ or } x = 4.$$

The area is given by the definite integral

$$\int_0^4 \sqrt{x} - x/2 \, dx = \frac{2}{3}x^{3/2} - \frac{x^2}{4}\Bigg]_0^4 = 16/3 - 4 = 4/3.$$