Math 1A | Calculus | Fall, 2004

Prof. Haiman

Quiz 12 Solution (Version A)

1. Evaluate the indefinite integral

$$\int x\sqrt{1+x}\,dx.$$

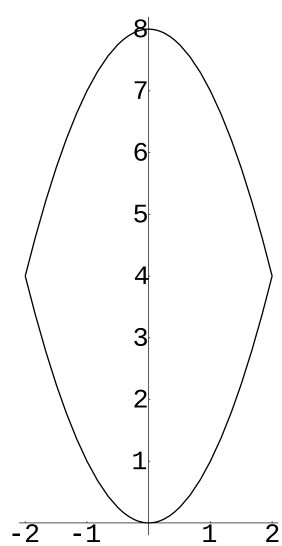
Substitute $u = 1+x$, $du = dx$:

$$\begin{aligned}\int x\sqrt{1+x}\,dx &= \int (u-1)\sqrt{u}\,du\\ &= \int u^{3/2} - u^{1/2}\,du\\ &= \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C\\ &= \frac{2}{5}(1+x)^{5/2} - \frac{2}{3}(1+x)^{3/2} + C.\end{aligned}$$

2. Sketch the region enclosed by the curves $y = x^2$, $y = 8 - x^2$, and find its area.

Find the endpoints by solving

$$x^2 = 8 - x^2 \quad\Rightarrow\quad 2x^2 = 8 \quad\Rightarrow\quad x^2 = 4 \quad\Rightarrow\quad x = \pm 2.$$



The area is given by the definite integral

$$\int_{-2}^{2} 8 - 2x^2\,dx = 8x - \frac{2}{3}x^3\Big]_{-2}^{2} = (16 - 16/3) - (-16 + 16/3) = 64/3.$$