Quiz 10 Solution (Version B)

1. Evaluate the integral by interpreting it in terms of areas. (Your solution should clearly indicate how to obtain the answer using areas, even if you already know another method of evaluating this integral.)

$$\int_{-2}^{2} \sqrt{4 - x^2} \, dx$$

The region under the graph of $y = \sqrt{4 - x^2}$, or $x^2 + y^2 = 4$, is a semicircle of radius 2. Since the interval of integration is $[-2, 2]$, we get the full area of this semicircle, which is $(1/2)\pi \cdot 2^2 = 2\pi$.

2. Verify the inequality, without evaluating the integral.

$$2/5 \le \int_{2}^{4} \frac{1}{1 + x} \, dx \le 2/3.$$

On the interval $[2, 4]$, we have $1/5 \le 1/(1 + x) \le 1/3$. The length of the interval is 2. Therefore the integral is between $2(1/5) = 2/5$ and $2(1/3) = 2/3$.