

Quiz 10 Solution (Version A)

1. Evaluate the integral by interpreting it in terms of areas. (Your solution should clearly indicate how to obtain the answer using areas, even if you already know another method of evaluating this integral.)

$$\int_1^6 |x - 3| dx$$

The region under the graph of $y = |x - 3|$ from $x = 1$ to $x = 3$ is a right triangle with height 2, base 2, and area $A_1 = 4/2 = 2$. The region from $x = 3$ to $x = 6$ is a right triangle with height 3, base 3, and area $A_2 = 9/2$. Therefore,

$$\int_1^6 |x - 3| dx = A_1 + A_2 = 13/2.$$

2. Verify the inequality, without evaluating the integral.

$$\pi/6 \leq \int_0^{\pi/3} \cos x dx \leq \pi/3.$$

On the interval $[0, \pi/3]$, we have $1/2 \leq \cos x \leq 1$. The length of the interval is $\pi/3$. Therefore the integral is between $(1/2)(\pi/3) = \pi/6$ and $1(\pi/3) = \pi/3$.