

Math 185—Introduction to Complex Analysis  
Haiman, Summer 2014

**Problem Set 8**

Due Monday, August 11

This problem set covers material from Chapters 8 and 9 which we will discuss in class the week of August 4-7.

The applications in Chapter 10 will be discussed in class on Monday and Tuesday, August 11-12. I will assign one last problem set on the Chapter 10 material, to be due on Wednesday, August 13.

The Final Exam will be held in class on Thursday, August 14, covering all topics on which homework has been assigned, with greater emphasis on material not already covered on the Midterm.

On Wednesday I plan to discuss a special topic—the Riemann zeta function—which I think you will find interesting, but for which you will not be responsible on the final exam.

Exercises from the textbook:

98.12  
100.2, 100.6, 100.7, 100.12  
102.7  
106.9  
108.1, 108.6  
114.1, 114.6(b), 114.8, 114.10  
115.4  
117.2, 117.4, 117.5, 117.6

Additional problems:

1. Find a linear transformation that maps the square with corners  $0$ ,  $1$ ,  $i + 1$ ,  $i$  onto itself, while rotating it through an angle  $\pi/2$ , so that each corner maps to the next one in the order I listed them.

2. Show that the transformation  $w = z + 1/z$  maps both the upper and lower halves of the circle  $|z| = 1$  onto the interval  $[-2, 2]$  on the real axis, and maps the portions  $[1, \infty)$  and  $(-\infty, -1]$  of the real axis in the  $z$  plane onto the portions  $[2, \infty)$  and  $(-\infty, -2]$  of the real axis in the  $w$  plane. Where does it map the interval  $[-1, 1]$  of the real axis in the  $z$  plane?

3. Find a harmonic conjugate  $v(x, y)$  of  $u(x, y) = \ln(x^2 + y^2) \tan^{-1}(y/x)$  and a concise expression for  $f(z) = u(x, y) + iv(x, y)$ , where  $z = x + iy$ , on a suitable domain.