Math 185—Introduction to Complex Analysis
Haiman, Summer 2014

Problem Set 7
Due Monday, August 4

Exercises from the textbook:

86.2, 86.8 (note that the integral is convergent at both ends, so its 'Cauchy principal value' is its actual value).

88.4, 88.9 (and justify the convergence of the integral), 88.12

90.1, 90.2, 90.3, 90.5

93.5

94.5, 94.8

Additional problems:

1. (a) Show that

$$\operatorname{Res}_{z=e^{i\pi/n}}\left(\frac{1}{z^n+1}\right) = \frac{-e^{i\pi/n}}{n},$$

for every positive integer n (see §83, Theorem 2).

(b) Use a contour with straight sides along the positive real axis and the ray at angle $2\pi/n$, and a circular arc of large radius R connecting them (see Exercise 86.9 for an illustration when $n = 3$) to show that

$$\int_0^\infty \frac{1}{x^n+1}\,dx = \frac{\pi}{n\sin(\pi/n)}$$

for all integers $n \geq 2$.

2. (a) Show that the equation $z = e^{-z}$ has exactly one real solution $z = x_0$.

(b) Use Rouché's Theorem to show that $z = e^{-z}$ has no other solutions in the half-plane $\operatorname{Re}(z) \geq x_0$.