Math 185—Introduction to Complex Analysis
Haiman, Summer 2014

Problem Set 6
Due Monday, July 28

Exercises from the textbook:

29.5
65.4, 65.8
68.1, 68.3, 68.5, 68.9
72.1, 72.2, 72.3, 72.4, 72.6, 72.7, 72.11
73.4 (and explain why $0 < |z| < 2\pi$ is the domain of convergence)
77.1(e), 77.2(c), 77.4(a), 77.7
79.1, 79.3
81.3(b)
83.4(a), 83.5(a)

Additional problems:

1. Prove the Newton binomial theorem

$$(z+1)^a = \sum_{n=0}^{\infty} \binom{a}{n} z^n \quad \text{for } |z| < 1,$$

where a is any complex number. Here we define

$$\binom{a}{n} = \frac{a(a-1)\cdots(a-n+1)}{n!},$$

and we use the principal value $(z+1)^a = e^{a \operatorname{Log}(z+1)}$.

2. (a) The Bessel function

$$J_0(z) = \frac{1}{\pi} \int_0^{\pi} \cos(z \sin\phi)\, d\phi$$

was defined in Exercise 68.9, where it was also shown to be the coefficient of w^0 in the Laurent series representation of $e^{z(w-1/w)/2}$. By writing $e^{z(w-1/w)/2}$ as the product $e^{zw/2}e^{-z/(2w)}$, find the Maclaurin series of $J_0(z)$.

(b) Using part (a) and the Maclaurin series of $\cos(z \sin\phi)$ as a function of z, find a formula for

$$\frac{1}{\pi} \int_0^{\pi} (\sin\phi)^{2n}\, d\phi.$$

3. (a) Establish the series representation

$$\operatorname{Log}(z) = \sum_{n=1}^{\infty} \frac{(1-1/z)^n}{n} \quad \text{for } \operatorname{Re}(z) > 1/2.$$

(b) Express $\ln(3)$ as the sum of a series of rational numbers.