Math 185—Introduction to Complex Analysis
Haiman, Summer 2014

Problem Set 3
Due Monday, July 7

Exercises from the textbook:

18.1(c), 18.5, 18.9, 18.11
20.3, 20.4, 20.8(a), 20.9
24.2(d), 24.4(b), 24.6, 24.7
26.1(c), 26.2(b), 26.7
27.2, 27.3

Additional problems:

1. Find
$$\lim_{z \to -1+i} \frac{(1+i)z+2}{z^2+2z+2}.$$

2. Prove that $f(z) = 1/z$ is continuous for all $z \neq 0$. Hint: the hardest way is to work directly from the definition. Try one of the following alternatives: (a) use Theorem 18.3 from the textbook (b) use $1/z = \overline{z}/|z|^2$ and the fact that the real-valued function $1/x$ is continuous for $x \neq 0$, or (c) prove the stronger assertion that $f(z)$ is differentiable for all $z \neq 0$.

3. Exercise 26.6 with the following modification: take the domain of definition of $g(z) = \ln r + i\theta$ to be $(r > 0, -\pi < \theta < \pi)$ instead of $(r > 0, 0 < \theta < 2\pi)$, and then show that $G(z) = g(z^2+1)$ is analytic in the whole right half-plane $x > 0$, with $G'(z) = 2z/(z^2+1)$.