Math 185—Introduction to Complex Analysis
Haiman, Summer 2014

Problem Set 2
Due Monday, June 30

Exercises from the textbook:

5.4
6.10, 6.11, 6.14
9.2, 9.9
11.4
12.1–12.6
14.3, 14.4, 14.6, 14.8(a,b).

Additional problems:

1. Recall that the n-th roots of 1 (including $z_0 = 1$) are $z_0, z_1, \ldots, z_{n-1}$, where $z_k = e^{2\pi ik/n}$.

(a) Prove algebraically that $z_0 + z_1 + \cdots + z_{n-1} = 0$ (assuming $n > 1$). For a hint, consult Exercise 11.7 in the textbook.

(b) Give a geometric explanation of the identity in part (a).

(c) Letting $w = e^{2\pi i/5}$, use the identity in part (a) to show that the real number $x = w + \overline{w}$ is a root of the quadratic equation $x^2 + x - 1 = 0$.

(d) Show that the other root of the quadratic equation in part (c) is $w^2 + \overline{w}^2$.

(e) Express $\cos(\pi/5)$ and $\cos(2\pi/5)$ in terms of the roots of the equation in (c), then use this to find exact formulas for $\cos(\pi/5)$ and $\cos(2\pi/5)$ involving rational numbers and $\sqrt{5}$.

2. The textbook supplies answers to two parts of Exercise 14.1, but one of them is incorrect. Which answer is wrong, and which part of Exercise 14.1 is it the right answer to?