Prof. Haiman Spring 2012

Math 110 Assignment 6

(I) *Exercises.*

Axler Chapter 5: 4, 6 (hint: what is the matrix of T in the standard basis?), 7 (hint: find the nullspace of T), 8, 11, 17, 18, 19, 20, 23, 24

(a) Show that the map $S\colon P_3(\mathbb{R}) \to \mathbb{R}^4$ defined by $Sp(z) = (p(0), p(1), p'(0), p'(1))$ is linear and injective. Hint: show that $p(a) = p'(a) = 0$ if and only if $(z-a)^2$ is a factor of $p(z)$.

(II) *Problems.* Due Friday, Mar. 9 by 3pm at the location your GSI has specified for turning in homework.

1. (4/10) Axler Chapter 5, Exercise 21

2. (6/10) Suppose V is finite-dimensional and $P \in \mathcal{L}(V)$. Prove that the following are equivalent:

(a) $P^2 = P$,

(b) P is diagonalizable and its eigenvalues belong to the set $\{0, 1\}$,

(c) there exist subspaces $U, W \subseteq V$ such that $V = U \oplus W$ and $P = P_{U,W}$, *i.e.*, P is the projection operator on the direct summand U.