Prof. Haiman Spring 2012

Math 110 Assignment 2

(I) *Exercises.*

Axler Chapter 2: 1, 2, 5, 6, 8, 9, 11, 13, 16, 17

(a) Prove that for a list of vectors $(v_1, \ldots, v_n)$ in V, the following three conditions are equivalent:

(i) $(v_1, \ldots, v_n)$ is linearly dependent;

(ii) one of the vectors is in the span of the previous ones on the list;

(iii) one of the vectors is in the span of all the others on the list.

For the next exercises, recall that when X is a set of vectors in V (as opposed to a list, and not necessarily assuming X is finite), we define $\mathrm{span}(X)$ to be the set of vectors which are linear combinations of some finite list of vectors in X, and we define X to be linearly independent if every finite list of *distinct* vectors in X is linearly independent.

(b) Show that for all lists $(v_1, \ldots, v_n)$ of vectors in V, we have $\mathrm{span}(v_1, \ldots, v_n) = \mathrm{span}(\{v_1, \ldots, v_n\})$.

(c) Show that for all lists $(v_1, \ldots, v_n)$ of vectors in V, the list $(v_1, \ldots, v_n)$ is linearly independent if and only if its members are all distinct and the set $\{v_1, \ldots, v_n\}$ is linearly independent.

(d) Show that for any set X of vectors in V, $\mathrm{span}(X)$ is the smallest subspace of V which contains X.

(e) Show that for any set X of vectors in V, X is linearly independent if and only if, for all distinct vectors $v_1, \ldots, v_n \in X$, and sclars $a_i \in \mathbb{F}$,

$$a_1 v_1 + \cdots + a_n v_n = 0 \quad \text{implies} \quad a_1 = \cdots = a_n = 0.$$

(II) *Problems.* Due Friday, Feb. 3 by 3:00pm at your GSI's office or mailbox.

Axler Chapter 2: 7, 14