Matrix Computations & Scientific Computing Seminar

Organizer: James Demmel & Ming Gu

Wednesday, 12:00–1:00PM, 380 Soda

Sept. 07 **Gautam Wilkins**, UC San Diego

Finding zeros of single-variable real-valued functions

Finding zeros of single-variable, real-valued functions is a common and basic task in scientific computing. Given an interval $[a, b]$, and a single-variable, real-valued function, f, that is continuous on $[a, b]$, if $f(a)f(b) < 0$, then it is guaranteed that $f(x) = 0$ for at least one $x \in [a, b]$. The task of finding a zero of a function is in general twofold. First, we find an interval, $[a, b]$, across which the function f reverses sign (also called a straddle). Second, we must reduce the size of the interval $[a, b]$ until it is sufficiently small to bracket a zero of f.

Given a straddle, we can use Brent's Method, which is one of the most popular methods for finding zeros of functions. This method usually converges very quickly to a zero; for the occasional difficult functions encountered in practice, it typically takes $O(n)$ iterations to converge, where n is the number of steps required for the bisection method to find the zero to approximately the same accuracy. While it has long been known that in theory Brent's Method could require as many as $O(n^2)$ iterations to find a zero, such behavior had never been observed in practice. This presentation will first show that Brent's Method can indeed take $O(n^2)$ iterations to converge, by explicitly constructing such worst case functions, and also show how a simple modification can reduce the worst case complexity to $O(n)$. It will then show how this approach can be generalized to ensure that any zero-finding method that is locally super-linearly convergent can use a straddle to converge in at worst $O(n)$ time.

This presentation will also discuss the computational task of finding a straddle for a function, focusing on the method used by Matlab's `fzero` function, and introduce a new method that is more robust.