UCB Math 228A, Fall 2014: Homework Set 6

Due December 12, 2014

1. Solve the integral equation

$$\sigma(x) + \int_\Gamma \mathcal{K}(x,y)\sigma(y)dS_y = f(x),$$

where $x = (x_1, x_2)^T, y = (y_1, y_2)^T \in \mathbf{R}^2$ are points defined on the unit circle Γ centered at 0, and the kernel function is

$$\mathcal{K}(x,y) = \mathbf{log}(\|x - y\|), \quad \|x - y\| = \sqrt{|x_1 - y_1|^2 + |x_2 - y_2|^2}.$$

You should divide Γ into $M = 4$ segments, and use Legendre Quadrature with $n = 5, 10, 15, 20$ nodes. Assuming that $n = 20$ gives the exact solution, estimate the errors in the solutions with $n = 5, 10, 15$. We will choose $f(x) = 1$.

Code Submission: E-mail all requested and supporting MATLAB files to Luming at `lwang@berkeley.edu` as a zip-file named lastname_firstname_6.zip.