UCB Math 228A, Fall 2014: Homework Set 5

Due Nov. 24, 2014

Code Submission: E-mail all requested and supporting MATLAB files to Luming at `lwang@berkeley.edu` as a zip-file named lastname_firstname_5.zip, for example luming_wang_5.zip.

1. Write a MATLAB function of the form

```
function [p,t,e]=pmesh(pv,hmax,nref)
```

which generates an unstructured triangular mesh of the polygon with vertices pv, with edge lengths approximately equal to $h_{\max}/2^{n_{\text{ref}}}$, using a simplified Delaunay refinement algorithm. The outputs are the node points p (N-by-2), the triangle indices t (T-by-3), and the indices of the boundary points e.

(a) The 2-column matrix pv contains the vertices x_i, y_i of the original polygon, with the last point equal to the first (a closed polygon).

(b) First, create node points along each polygon segment, such that all new segments have lengths $\leq h_{\max}$ (but as close to $h_{\max}$ as possible). Make sure not to duplicate any nodes.

(c) Triangulate the domain using the delaunayn command.

(d) Remove the triangles outside the domain (see the inpolygon command).

(e) Find the triangle with largest area A. If $A > h_{\max}^2/2$, add the circumcenter of the triangle to the list of node points.

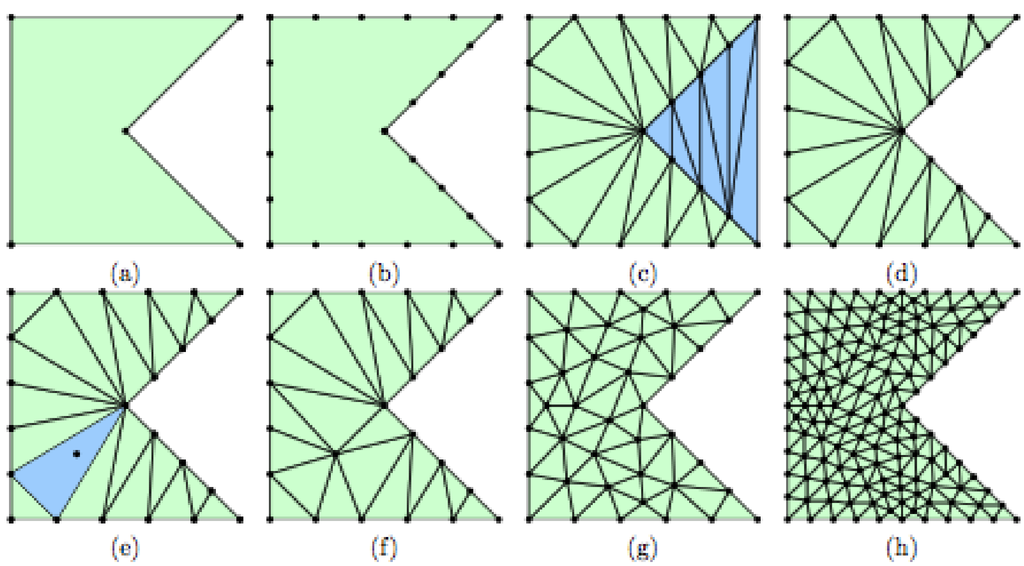
(f) Retriangulate and remove outside triangles (steps (c)-(d)).

(g) Repeat steps (e)-(f) until no triangle area $A > h_{\max}^2/2$.

(h) Refine the mesh uniformly n_{ref} times. In each refinement, add the center of each mesh edge to the list of node points, and retriangulate. Again, make sure not to duplicate any nodes, see e.g. the command unique(p,'rows').

Finally, find the nodes e on the boundary using the boundary_nodes command. The following commands create the example in the figures. Also make sure that the function works with other inputs, that is, other polygons, $h_{\max}$, and n_{ref}.

```
pv=[0,0;1,0;.5,.5;1,1;0,1;0,0];
[p,t,e]=pmesh(pv,0.2,1);
tplot(p,t)
```



(a) (b) (c) (d) (e) (f) (g) (h)

2. Implement a MATLAB function

```
function u=fempoi(p,t,e)
```

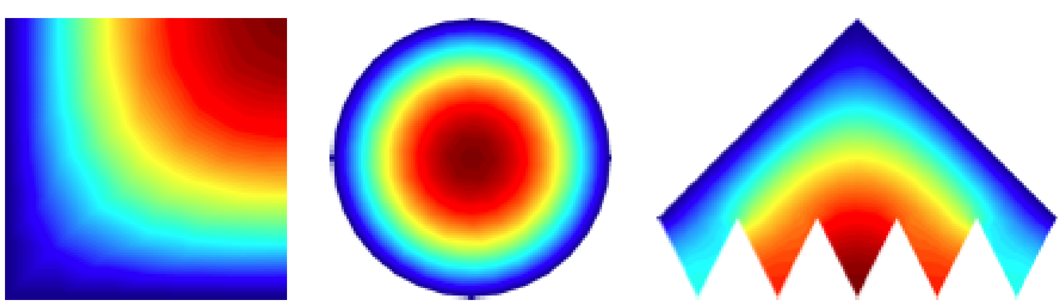
that solves Poissons's equation $-\nabla^2 u(x,y) = 1$ on the domain described by the unstructured triangular mesh `p,t`. The boundary conditions are homogeneous Neumann ($n \cdot \nabla u = 0$) except for the nodes in the array `e` which are homogeneous Dirichlet ($u = 0$).

Here are a few examples for testing the function.

```
% Square, Dirichlet left/bottom
pv=[0,0;1,0;1,1;0,1;0,0];
[p,t,e]=pmesh(pv,0.2,0);
e=e(p(e,1)==0 | p(e,2)==0);
u=fempoi(p,t,e);
tplot(p,t,u)

% Circle, all Dirichlet
n=32; phi=2*pi*(0:n)'/n;
pv=[cos(phi),sin(phi)];
[p,t,e]=pmesh(pv,2*pi/n,0);
u=fempoi(p,t,e);
tplot(p,t,u)

% Complex polygon geometry, mixed Dirichlet/Neumann
x=(0:.1:1)';
y=.1*cos(10*pi*x);
pv=[x,y; .5,.6; 0,.1];
[p,t,e]=pmesh(pv,0.05,0);
e=e(p(e,2)>=.6-abs(p(e,1)-.5));
u=fempoi(p,t,e);
tplot(p,t,u)
```



3. Implement a MATLAB function

```
function errors=poiconv(pv,hmax,nrefmax)
```

that solves the all-Dirichlet Poisson problem for the polygon `pv`, using the mesh parameters `hmax` and `nref=0,1,...,nrefmax`. Consider the solution on the finest mesh the exact solution, and compute the max-norm of the errors at the nodes for all the other solutions (note that this is easy given how the meshes were refined – the common nodes appear first in each mesh). The output `errors` is a vector of length `nrefmax` containing all the errors.

Test the function using the commands below:

```
hmax=0.15;
for pv={[0,0;1,0;1,1;0,1;0,0],[0,0;1,0;.5,.5;1,1;0,1;0,0]}
  errors=poiconv(pv{1},hmax,3)
  loglog(hmax./2.^(0:2),errors)
  rate=log2(errors(end-1))-log2(errors(end))
end
```