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Sample Midterm I, Brief Solutions

This is a closed book, closed notes exam. You need to justify every one of your answers unless you are asked not to do so. Completely correct answers given without justification will receive little credit. Look over the whole exam to find problems that you can do quickly. You need not simplify your answers unless you are specifically asked to do so. Hand in this exam before you leave.

Problem	Maximum Score	Your Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

Your Name & SID: ______________________________

Your Section & GSI: ______________________________

1. Find the points of intersection of

   (a) $y = x^2 - 5x$ and $y = -x - 3$.
   Solution: $x = 1, 3$.

   (b) $y = -\dfrac{1}{x+1}$ and $y = x + 3$.
   Solution: $x = -2$.

2. Compute the derivatives of the following functions

   (a) $f(x) = x\left(\dfrac{1}{\sqrt{x}} - x^{\frac{2}{3}}\right)$.
   Solution: $f(x) = \sqrt{x} - x^{\frac{5}{3}}$. Use power rules in the book.

   (b) $f(x) = \dfrac{1}{\sqrt{x^2+1}}$. Solution: $f'(x) = -\frac{x}{(x^2+1)^{3/2}}$.

3. Compute the following limits

   (a) $\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}$.
   SOLUTION: $x-1 = (\sqrt{x}-1)(\sqrt{x}+1)$. So
   $$\frac{\sqrt{x}-1}{x-1} = \frac{1}{\sqrt{x}+1}.$$
   Limit is 1/2.

   (b) $\lim_{x\to\infty} \frac{x^2-1}{x^3+1}$.
   SOLUTION: Limit is 0.

4. Compute the second derivatives of the following functions

   (a) $f(x) = x^4 + x^3 + x^2 + 1$.
   SOLUTION: $f''(x) = 12x^2 + 6x + 2$.

   (b) $f(x) = \dfrac{1}{(x+1)^2}$.
   SOLUTION: $f''(x) = \frac{6}{(x+1)^4}$.

5. Using the limit definition of the derivative to compute the derivative of $f(x) = \dfrac{1}{\sqrt{x+1}}$ at $x = 1$.

   SOLUTION: Since
$$\frac{\sqrt{2+h}-\sqrt{2}}{h} = \frac{1}{\sqrt{2+h}+\sqrt{2}},$$
   we have
$$\begin{aligned}
\lim_{h\to 0}\frac{f(1+h)-f(1)}{h} &= \frac{\frac{1}{\sqrt{2+h}}-\frac{1}{\sqrt{2}}}{h} = -\lim_{h\to 0}\frac{\sqrt{2+h}-\sqrt{2}}{h\sqrt{2+h}\sqrt{2}} \\
&= -\lim_{h\to 0}\frac{1}{\sqrt{2+h}\sqrt{2}(\sqrt{2+h}+\sqrt{2})} \\
&= -\frac{1}{4\sqrt{2}}.
\end{aligned}$$

6. Sketch the graph of the function $f(x) = |x-1| + |x+1|$. Discuss the continuity and differentiability of $f(x)$ at points $x = -1, 0, 1$.

   SOLUTION: Continuous at $x = -1, 0, 1$; differentiable at $x = 0$ but not $x = -1, 1$.

7. Find all equations of the lines of the form $y = mx - 1$ that are tangent to the curve $y = x^2 - 2x$.
   SOLUTION: Since $y'(x) = 2x - 2$, let $(a, a^2 - 2a)$ be the point where the line and the curve intersect, and let $2a - 2$ be the slope. By the point-slope form of the line, we can write the tangent line as $y - (a^2 - 2a) = (2a - 2)(x - a)$, which simplifies to $y = (2a - 2)x - a^2$. Since the line has to be of the form $y = mx - 1$, we have $m = 2a - 2$ and $a^2 = 1$. This leads to $a = 1$ $(m = 0)$ and $a = -1$ $(m = -4.)$

8. Suppose that \$2000 is invested at an interest of 10% per year, compounded twice per year. What is the the compound interest at the end of the first year?
   SOLUTION: the compound interest $= 2000 * ((1 + 0.1/2)^2 - 1)$.

9. Consider the cost function $C(x) = 1 + x + \frac{1}{x+1}$ and revenue function $R(x) = 2x + x^2$. Compute the marginal cost, marginal revenue and profit at $x = 3$.
SOLUTION: $C'(3) = 1 - \frac{1}{4^2}$, $R'(3) = 8$ and $P(3) = R(3) - C(3)$.

10. Given functions $f(x)$ and $g(x)$, and assume that $\lim_{x\to a} f(x) = F$ and $\lim_{x\to a} g(x) = G$ exist. Show that $\lim_{x\to a}(f(x) - g(x)) = F - G$.
SOLUTION: Use Limit Theorems (I) and (III),

$$\lim_{x\to a}(f(x) - g(x)) = \lim_{x\to a}(f(x) + (-g(x))) = \lim_{x\to a} f(x) + \lim_{x\to a}(-g(x)) = F - \lim_{x\to a} g(x) = F - G$$