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Math128B: Numerical Analysis

Programming Assignment #4, Due May 4, 2009

In classical mechanics, the two-body problem is to determine the motion of two point particles that interact only with each other. An example is the earth orbiting the sun.

Let $\mathbf{x}_1$ and $\mathbf{x}_2$ be the positions of the two bodies, the Earth, and the Sun, and let m_1 and m_2 be their masses. To determine the trajactories $x_1(t)$ and $x_2(t)$ for time t, given initial positions $\mathbf{x}_1(t == 0)$, $\mathbf{x}_2(t == 0)$, and initial velocities $\mathbf{v}_1(t == 0)$, $\mathbf{v}_2(t == 0)$. According to Newton's second law,

$$F_{12}(\mathbf{x}_1, \mathbf{x}_2) = m_1 \ddot{\mathbf{x}}_1,$$
$$F_{21}(\mathbf{x}_1, \mathbf{x}_2) = m_2 \ddot{\mathbf{x}}_2,$$

where $F_{12}(\mathbf{x}_1, \mathbf{x}_2)$ is the force on Earth due to its interaction with the Sun; and $F_{21}(\mathbf{x}_1, \mathbf{x}_2) = -F_{12}(\mathbf{x}_1, \mathbf{x}_2)$ is the force on the Sun due to its interaction with Earth.

We assume that

- Earth and Sun are both on a plane, so that $\mathbf{x}_1$ and $\mathbf{x}_2$ have two components each.
- $$F_{12}(\mathbf{x}_1, \mathbf{x}_2) = \frac{\gamma m_1 m_2}{r^3} (\mathbf{x}_2 - \mathbf{x}_1),$$

  where $r = \|\mathbf{x}_2 - \mathbf{x}_1\|$.

On the class website, you can download a program, `SunEarthSat.m`, which simulates the orbits of three bodies: The Sun, the Earth, and an Earth Satelite. The constants used in this program are all true values, obtained from relevant wikipedia pages.

In this project, we should modify this programm to do the following:

- Use an 8^{th} order Anadromic scheme instead of the matlab function `ode45` for orbit calculations.
- Perform orbit calculations with 4 bodies: the Sun, the Earth, the Moon, and Jupiter. You can get from wikipedia the relevant constants: masses, distances, and velocities. These celestrial bodies will not stay together if given the wrong constants.

- Your output should be 5 plots which show:
  - the orbits of all 4 bodies;
  - the orbit of the Moon relative to the Earth;
  - the orbit of Earth relative to the Sun;
  - the orbit of Jupiter relative to the Sun;
  - the orbit of the Sun;

Programming Assignment #4 is due on May 4.