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Math128B: Numerical Analysis Sample Final Exam

This is a closed book, closed notes exam. You need to justify every one of your answers. Completely correct answers given without justification will receive little credit. Do as much as you can. Partial solutions will get partial credit. Look over the whole exam to find problems that you can do quickly.

Problem	Maximum Score	Your Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Your Name: ______________________________

Your SID: ______________________________

1. Let $A = \begin{pmatrix} 1 & 1 \\ 2 & 4 \end{pmatrix}$.

   (a) Compute the 1-norm and ∞-norm of A.

   (b) For the linear system of equations $Ax = b$ where $b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, define the Gauss-Seidel method.

   (c) Find out whether the Gauss-Seidel method converges for any initial guess x_0.

2. Given a weight function $w(x) = e^{-x}$.

   (a) Find constant and linear polynomials $p_0(x)$, $p_1(x)$ that are orthogonal on $[0, \infty)$ with respect to $w(x)$.

   (b) Let $f(x) = \cos x$. Find the best linear approximation to $f(x)$ for the given weight.

3. The nonlinear system

$$
\begin{aligned}
4x_1 - x_2 + x_3 &= x_1 x_4, \\
-x_1 + 3x_2 - 2x_3 &= x_2 x_4, \\
x_1 - 2x_2 + 3x_3 &= x_3 x_4, \\
x_1^2 + x_2^2 + x_3^2 &= 1
\end{aligned}
$$

has six solutions.

(a) Show that if $(x_1, x_2, x_3, x_4)^T$ is a solution then $(-x_1, -x_2, -x_3, x_4)^T$ is a solution.

(b) Show how to solve the nonlinear system via the computation of all the eigenvalues and eigenvectors of a 3×3 matrix.

4. Consider the boundary-value problem

$$y'' + y = 0, \quad 0 \le x \le b, \quad y(0) = 0, \quad y(b) = B.$$

Find choices for b and B so that the boundary-value problem has

- No solution
- Exactly one solution
- Infinitely many solutions.

5. Let $a \in \mathbf{R}^n$ be a non-zero vector. Develop a numerically stable procedure to compute a Householder transformation P such that

$$Pa = \begin{pmatrix} \|a\|_2 \\ 0 \end{pmatrix}.$$